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Abstract: This paper develops the analytical results essential to understand the literature on accounting data -- earnings, book values and dividends -- and equity value. The paper has been structured around eight propositions which try to achieve an as coherent flow of ideas as possible. The analysis starts by stating benchmark settings and results, and it then proceeds by introducing concepts that enrich the models' empirical content. Two themes recur throughout. First, the analysis distinguishes between book value vs. earnings oriented models though earnings take on the more prominent role. Second, to identify the relevant growth constructs one can focus on the performance measures that determine the price to forward-earnings ratio.

I. Introduction and Summary

Subsequent to Feltham and Ohlson (1995) and Ohlson (1995), the accounting literature has published a large numbers of papers on accounting data and value.[1] A review of this literature reveals that many themes and insights reoccur across the papers. With the advantage of hindsight, the repetitions seem inefficient. A student who takes a stab at familiarizing herself with subject matter naturally tends to view such a state of affairs as less than ideal. Questions like "What is it that I really need to understand?" or "Taken in its totality, what ideas and results make the literature tick?" arise. This paper addresses the essence of such questions. It states the central results as simple formal propositions (there will be eight). Because all the propositions are free-standing, they can, at least in principle, be internalized independently of each other. But the sequencing is in fact relevant because it introduces step-by-step increasingly sophisticated concepts. The concepts build upon each other; the propositions' analytical simplicity should therefore not be taken as a sign that they are conceptually simplistic. Much discussion follows the propositions to spell out their broader significance. And the paper approaches this task always maintaining the texture of accounting: the central variables are earnings, book values, and dividends.

The paper does not digress on proofs and finer analytical points. These aspects are of little interest, which is another way of saying that the paper focuses squarely on analytical constructs/representations and how these fit together. Nor does this paper elaborate on the extent to which the literature has already dealt with the results or insights. There is no question that most, if not all, results have had some kind presence and thus lack novelty. That said, such cataloging and related discussion would have been long and tedious without facilitating a better understanding of the insights I wish to convey.

Aside from trying to systemizing the literature, and thereby making it more accessible to the average reader, the paper also has a more ambitious objective. It goes to the heart of subject matter: the exposition should give the reader the sense that all pieces and insights forge tight links so that the totality coalesces into a whole. In other words, the task at hand is to go beyond a listing of useful results (though this should hopefully be the case too). The development of such a coherent mental map allows the reader to think of the broad literature in an integrated fashion, rather than as consisting of loosely connected, or even competing, models that primarily differ in their empirical content. Only with an integrated, yet broad, overview can a reader compare how models differ in certain key characteristics (such as the relevance of certain earnings growth measures). Such evaluations set the stage for making judgments whether a model is likely to be useful in empirical or practical applications.

Papers of my own (often co-authored) that capture the results in this paper the most are Ohlson (1992, 1995, 2006), Feltham and Ohlson (1995, 1996), Ohlson and Jeuttner-Nauroth (2006), Ohlson and Gao (2006). This paper also picks up significant ideas and results found in Peasnell (1981), Ryan (1986), Yee (2005, 2006), and Ozair (2003).

II. Topics Covered

This section outlines the topics in the order which they will be discussed. A reader who is reasonably familiar with the literature will get an overview as to what the paper deals with. Less versed readers have to be more patient because topic descriptions may seem terse and obscure as to what they refer to. Some readers may even decide to skip this section and tackle the topics sequentially without much concern as to what may follow next. At any rate, broadly speaking the paper can be split into two halves, each covering four propositions. The first half deals with highly stylized and parsimonious models: a savings account, permanent earnings and economic earnings. A full appreciation of these models lays an indispensable foundation for the much richer models the second half develops. This first half also tries to motivate the questions that ought to be addressed. Reading the first half therefore helps to come to grips with why certain assumptions and issues recur even when one wants to articulate models with empirical content. In the second half the results relate more to what is taught in classrooms and frameworks/models that have been used in empirical research (e.g., Residual Income Valuation, or RIV).

The next two sections consider the nature of earnings for a savings (bank) account in a certainty setting. Because many subsequent insights depend on understanding the analytics of this earnings construct, the discussion of the savings account will be quite extensive (perhaps surprisingly so). Of particular interest will be the time series behavior of earnings and how it ties in with the price to forward earnings ratio. It lays the foundation for a topic of great interest through out the paper, namely how one conceptualizes the price to forward earnings ratio. Following the savings account, the paper introduces earnings constructs that embed uncertainty. With this added feature the analysis posits the two well-known benchmark models of earnings, permanent earnings and economic earnings. In the former case earnings explain value itself, whereas in the latter it explains the change in value. In turn this means that the relevant information for forecasting purposes is either earnings or book value. While the two models are obviously different, they in fact have much in common. Dividend policy irrelevancy is one common property. It is also shown that in both cases one can equate price to the capitalization of next period's expected earnings (and where of course price also equals the present value of expected dividends). And it turns out that there are no other benchmarks that satisfy this stringent criterion.

The second half of the paper considers settings in which price does *not* equal capitalized (expected) forward earnings. It sets the stage for the analysis of growth. A coherent set of results need to show the tight link between the price to forward earnings ratio and measures of growth in expected earnings or growth in expected book values. As a preliminary to deal with growth, the paper lays out the framework for (i) RIV or, more generally, Abnormal Book Value Growth (ABG) and (ii) Abnormal Earnings Growth (AEG). It is underscored that these two PVED-equivalent representations rest on the same analytical foundation. This equivalence brings out that the two representations differ only because one emphasizes book values while the other emphasizes earnings. RIV, of course, represents PVED via book value plus the PV of residuals earnings. But this derivation introduces earnings, via clean surplus accounting, in a redundant second step. On a more basic level residual earnings simply equal the change in book values with an adjustment for dividends. This way of looking at residual earnings leads to the ABG representation. Moving away from book values (and clean surplus), a direct earnings perspective obtains via AEG. It represents PVED via capitalized forward earnings plus the PV of capitalized increments in future expected earnings (adjusted for earnings due to retained earnings). While many readers may view RIV and AEG as practical valuation tools, my main purpose is to exploit the equivalent PVED representations to show how growth enters into the picture.

The more detailed study of growth assumes that the PV-variable (like residual earnings) satisfies a standard growth/decay dynamic. RIV then leads to the market-to book model familiar from many textbooks (e.g., Penman (2006)), and AEG leads to the so-called OJ model (Ohlson and Jeuttner-Nauroth (2005)). *Both* models can explain the price to forward earnings ratio, but they do so with different explanatory variables. In case of AEG and the OJ-model, the growth in expected earnings explains the price to forward earnings ratio. In case of RIV and the market-to-book model, the return on equity explains the price to forward-earning ratio. Thus one obtains two distinct ways of explaining the price to forward earnings ratio without changing the mathematics. With these insights in place one can next ask what kinds of information dynamics sustain the results. It is shown that to derive the relations one extends, respectively, the dynamics of economic earnings and permanent earnings. These extensions now allow for information about growth expectations. And with growth being expected it follows that the firm's price is at a premium relative to book values and capitalized forward earnings. In other words, growth and conservative accounting are the two sides of the same coin.

III. The Savings Account: General Considerations

Why study the analytics of a savings account? Can such an elementary model help us develop a theory of value and accounting data that captures and describes principles of real world equity valuation? Though any answers to these questions involve an element of taste, I think the study of a savings account helps for three reasons. To spell these out, which is what the remainder of this section does, should motivate why the study of a savings account serves as such an essential prelude to the study of the more complicated set of problems spanned by the title of this paper.

First, almost any theoretical development can benefit from an initial analysis of obviously unrealistic settings. One gets a flavor for the topic; it introduces those variables and parameters that are likely to be present no matter what direction the modeling takes. Specifically, the topic "accounting data and value" suggests that the modeling of accounting data must reflect the creation and distribution of value. It points toward earnings and dividends as being the central, indispensable variables. "What are the characteristics of earnings?" and "How do earnings differ from, yet align with, dividends?" are the obvious questions that must be addressed. There is no better way of getting a feel for the role of these two core variables than analyzing a savings account.

Second, a savings account setting directs the analysis to specific questions, which should be – and will be – of interest whenever one deals with equity value as a theoretical or practical matter:

- How does price relate to *future* expected outcomes of earnings and other accounting variables?
- How does price relate to *current* accounting data (and other kinds of information)?
- How does price relate to the intertemporal, time-series *dynamics* of accounting data?

Regardless of a valuation model's specific complexity, any analysis of the three questions leads to intertwined answers. A savings account illustrates this interconnection in a particularly poignant fashion.

Third, an analysis of simpler models often helps to appreciate the driving ideas inherent in a full-fledged model. These simple models bring out how parts and pieces of the full-fledged model derive as special cases with more confined sets of assumptions. The source and role of a model's more complicated features thereby come into sharper focus if the simpler models have already been internalized. "First things first" accordingly suggests that the formal analysis should start by looking at the most straightforward valuation setting, namely a savings account.

IV. The Savings Account: The Analytics

Throughout I use the following notation:

p_t = price, or value, of equity

x_t = earnings

d_t = (net) dividends

b_t = book value

$R = 1 + r (> 1)$ = the discount factor, an exogenous constant. Some settings depend on the clean surplus relation, CSR, $\Delta b_t + d_t = x_t$. Given CSR, define

$x_t^a \equiv x_t - rb_{t-1}$ = residual earnings.

As is standard, the analysis assumes that PVED determines value:

$$p_t = \sum_{r=1}^{\infty} R^{-\tau} E_t\left[\tilde{d}_{t+r}\right] \qquad \text{[PVED]}$$

Absent uncertainty, which applies in the preliminary analysis of a savings account, PVED reduces to PVD:

$$p_t = \sum_{r=1}^{\infty} R^{-\tau} d_{t+r} \qquad \text{[PVD]}$$

The first result articulates the multiple, but equivalent, ways that characterize earnings on a savings account. Understanding these properties of earnings helps because they give an intuitive feel for various conclusions in settings *with* uncertainty.

Proposition I: Assume PVD and consider the 3 statements

(i) $p_t = \dfrac{x_{t+1}}{r}$

(ii) $p_t = \dfrac{R}{r} x_t - d_t$

(iii) $x_{t+1} = Rx_t - rd_t$.

Given any one of the three statements, the two that remain follow.

Proving the proposition is trite, except, perhaps, for the claim that (iii) implies (i) and (ii). (Simple proofs of the various statements use that PVD is equivalent to $p_t + d_t = Rp_{t-1}$.)

Comments:

- The proposition shows that one identifies a savings account with a single crisp equation combined with PVD; any one of the three statements suffices. Additional implications of interest follow immediately. That said, for future reference it should be emphasized that (iii) is particularly interesting in so far that this earnings dynamic makes no reference to p_t. Still, from the dynamic one infers p_t, either in terms of (i) or (ii).
- While the notion of earnings capitalization is pervasive in valuation contexts, the savings account setting shows that a certain care is required. The capitalization depends on whether the focus is on current earnings, or on those forthcoming (in the next period). That is, the multiplier $R / r = 1 + 1/ r$ must be distinguished from $1/ r$. And in the former case the capitalization determines the cum-dividend value, $p_t + d_t = \left(R / r\right) x_t$.
- A savings account embeds dividend policy irrelevance, or DPI for short. This subtle property -- which takes on considerable significance more generally -- implies that the proposition requires no particular assumption on the dividend policy. One infers value without any knowledge about the *details* of the sequence of dividends. Yet PVD applies. For example, if one puts $d_t = Kx_t$, $K > 0$, $t \geq 1$, then the choice of K has *no* effect on p_0, although it affects the sequence $\{d_t\}$. Such a DPI conclusion goes beyond a fixed payout policy.[2] This DPI conclusion is surprisingly easy to miss yet true: in the proposition

neither (i) nor (ii) depends on any dividend policy parameters or the characteristics of the policy. Only a mild regularity condition is required, namely x_t / R^t as $t \to \infty$ (which also guarantees $d_t / R^t \to 0$ as $t \to \infty$). As time passes, wealth accumulated must be distributed at an adequate rate.

For a savings account book values and price coincide, $b_t = p_t$. It is readily shown that this result follows given any initialization $(x_0, d_0) = (0, -b_0)$ combined with Proposition I and CSR. Though the observation is trite in the current context, it will be of interest in a subsequent discussion of economic earnings.

The dynamic for earnings, (iii), can be restated as

$$x_{t+1} = x_t + r \cdot [x_t - d_t].$$

Expressing the dynamic this way makes it apparent that the change in earnings, $\vartriangle x_{t+1}$, depends only on the earnings retained in the current period. Accordingly, the dividend policy alone explains the growth in earnings. Zero growth corresponds to a 100% payout; a growth that equals the discount factor corresponds to a zero payout. Common sense suggests that this growth-effect due to retained earnings should influence any model of earnings and dividends. More general models, however, ought to also allow for growth that *goes beyond* this dividend policy effect, *unlike* a savings account.

Section VI deals with "superior" growth which goes beyond the growth due to retained earnings. Before dealing with this issue, the next section introduces uncertainty and related benchmark models of earnings and value.

V. Permanent Earnings and Economic Earnings

Using the savings account as a foundation, this section develops the two benchmark models: permanent earnings and economic earnings. Both of these models differ from the savings account in the previous section because they admit uncertainty. PVED accordingly replaces PVD. Critically, there is uncertainty about the creation of wealth -- the forthcoming period's earnings realization -- which in turn leads to uncertainty about the distribution of wealth or dividends. In this regard the two models are similarly motivated, though of course they have different valuation functions and information dynamics. Much of the discussion deals with these aspects, which can be thought of as generalizations of statements (ii) and (iii) in Proposition I. No less important are the properties that both models have in common. Of these, two are central. First, as a generalization of statement (i) in Proposition I, capitalized expected next-period earnings equal value. Second, both models satisfy DPI: all dividend policies have the same PVED, like a savings account.

The earnings dynamic (iii) in Proposition I motivates the first earnings construct to be discussed, permanent earnings. One expresses (iii) in expectation rather than for sure.[3,4] In other words, one simply adds a zero mean disturbance term to (iii) to obtain the dynamic for earnings:

$$
\begin{aligned}
\tilde{x}_{t+1} &= Rx_t - rd_t + \tilde{\varepsilon}_{t+1} \\
&= x_t + r\left(x_t - d_t\right) + \tilde{\varepsilon}_{t+1},
\end{aligned}
$$

where $E_t\left[\tilde{\varepsilon}_{t+r}\right]=0$ for all τ >1. (The disturbance term has no particular distribution, such as Gaussian normality, and its variance can be information dependent, e.g., heteroscedastic). The label "permanent earnings" communicates that earnings extrapolate in expectation, subject to an adjustment for (expected) earnings due to retained earnings.

By admitting uncertainty, the next proposition generalizes Proposition I. It also brings out that one can articulate a simple dividend policy - - similar in structure to the earnings dynamic - -

with *its* source of uncertainty without affecting any conclusions. Thus, the proposition below refers to a second source of uncertainty, ε_{2t}, as well as two dividend policy parameters, θ_1 and θ_2. Subject to minor regularity conditions, the result underscores that none of these aspects play any role though the analysis rests on PVED.

Proposition II: Assume PVED. Further assume the permanent earnings dynamic

$$\tilde{x}_{t+1} = Rx_t - rd_t + \tilde{\varepsilon}_{1t+1}$$

and a dividend dynamic

$$\tilde{d}_{t+1} = \theta_1 x_t + \theta_2 d_t + \tilde{\varepsilon}_{2t+1},$$

where $E_t\left(\tilde{\varepsilon}_{1t+\tau}\right) = E_t\left(\tilde{\varepsilon}_{2t+\tau}\right) = 0, \tau \geq 1$. Then, given the no-excess-growth regularity condition

max root $\begin{pmatrix} R & -r \\ \theta_1 & \theta_2 \end{pmatrix}$ < R

(i) $$p_t = \frac{R}{r} x_t - d_t$$

(ii) $$p_t = \frac{E_t\left[\tilde{x}_{t+1}\right]}{r}.$$

As a converse, PVED and the statement (i) imply the permanent earnings dynamic.

Comments:

- The conclusions are readily appreciated if one simply keeps in mind that the assumption $E_t\left[\tilde{\varepsilon}_{i,t+\tau}\right] = 0, \tau \geq 1,\ i = 1,2,$ implies that PVED reduces to the mathematics of PVD and the savings account.
- DPI applies because p_t does not depend on θ_1 and θ_2 (given x_t and d_t). Statement (i) makes the point crystal clear. To be sure, one can model much more general dividend

policies without affecting DPI and the conclusions of the proposition. DPI also means that the covariance between ε_{it} and ε_{2t} has no effect on the valuation.

- The condition on the 2×2 transition matrix is a mild regularity condition. It ensures convergence in PVED. (Neither $E_t\left[x_{t+\tau}\right]$ or $E_t\left[\tilde{d}_{t+\tau}\right]$ can now grow at a rate greater than R as $\tau\rightarrow\infty$). More subtly, the condition further implies that the distribution of wealth is consistent with its creation. It can be shown that the regularity condition holds only if $\theta_1 > 0$; on the margin and on the average, increased wealth creation thereby leads to increased wealth distribution.
- Whether there is uncertainty or not, the value function equals $p_t = (R\ /\ r)x_t - d_t$. As the last part of the proposition makes clear, this value function uniquely identifies the permanent earnings dynamic if one includes the savings account as a special case. One must, therefore, end up with permanent earnings, possibly with $\varepsilon_{2t} \equiv 0$, if one assumes $p_t = (R\ /\ r)x_t - d_t$ and PVED.
- With respect to forward earnings, uncertainty implies that one replaces the savings account relation $p_t = x_{t+1} / r$ with $p_t = E_t\left[\tilde{x}_{t+1}\right] / r$. In this way permanent earnings refers to a first cut, intrinsic valuation rule (which is of interest even in investment practice.)
- Permanent earnings does not by itself depend on CSR or a book value construct. But one can usefully check what happens if one adds CSR. Permanent earnings is now equivalent to

  $\tilde{x}^a_{t+1} = x^a_t + \tilde{\varepsilon}_{1,t+1}$.

  Regardless of the dividend policy, *residual* earnings follow a so-called random walk. Alternatively, $E_t\left[\Delta\tilde{x}^a_{t+\tau}\right] = 0$ so that the change in residual earnings is always a zero-mean random variable. This sharp characterization of permanent earnings can be compared with a savings account, which corresponds to $x_t^a = 0$. Clearly, the latter is a condition much sharper than the former. These are no surprises here. A more subtle point notes that there is another special case of $E_t\left[\Delta\tilde{x}_{t+\tau}^a\right] = 0$, namely $E_t\left[\tilde{x}_{t+\tau}^a\right] = 0$, which generalizes $x_t^a = 0$. This setting defines economic earnings. It will be developed later in this section.

- The preceding observations would be of no interest if they in any way they depended on restricting the dividend policy. But this is not necessary, of course. In this regard, since $\partial b_t / \partial d_t = -1$, and the dynamic implies that $\partial E_t\left[\tilde{x}_{t+1}\right] / \partial d_t = -r$, one obtains the dividend independence conditions $\partial E_t\left[\tilde{x}_{t+1}^a\right] / \partial d_t = 0$ and $\partial E_t\left[\Delta x_{t+1}^a\right] / \partial d_t = 0$.

Proposition II models two sources of uncertainty, ε_{1t} and ε_{2t}. The two sources take on distinct roles, and they differ in how they influence certain aspects of the model. Consider two mutually exclusive cases: (a) $\varepsilon_{2t} \equiv 0$ (for all *t*) while ε_{1t} remains random, and (b) its opposite so that $\varepsilon_{1t} \equiv 0$ while now ε_{2t} remains random. In case (a), the substantive aspects of the permanent earnings model hold. The second source of uncertainty, ε_{2t} can be zero but this fact is totally irrelevant because DPI applies. By contrast, for case (b) -- when ε_{1t} equals zero but ε_{2t} is random -- the model now reduces to a savings account in the sense that all aspects of Proposition I remain true. In other words, because ε_{2t} can be random, the assumptions of Proposition I have been relaxed without restricting the conclusions. Earnings are no longer certain since at date t x_{t+2}, x_{t+3},...$x_{t+\tau}$ are unknown due to the uncertainty caused by $\varepsilon_{2t+1},...\varepsilon_{2t+\tau-1}$. *But the uncertainty in* $\tau \geq 2$ *earnings caused by the uncertainty inherent in the dividend-policy is of no relevance.* We need the *value-creation* uncertainty, ε_{1t}, but not the value distribution uncertainty, ε_{2t}, to state permanent earnings as a model strictly more general than a savings account.

To further appreciate the significance of ε_{1t}, consider what happens if one puts $\varepsilon_{1t} = 0$ and defines $\tilde{p}(t,T) \equiv \sum_{\tau=1}^{T} R^{-r} \tilde{d}_{t+\tau}$. Though $\tilde{p}(t,T)$ is a random variable for every T due to $\tilde{\varepsilon}_{2t}$, $p(t,T)$ converges to the solution $\tilde{p}(t,\infty) = x_{t+1}/r$ with probability one as T goes to infinity (x_{t+1} is not random at date t because $\varepsilon_{1t+1} = 0$). It means the "E" in PVED is irrelevant though the dividends are random. From an ex post perspective, every possible ("infinitely long") realized sequence of dividends results in the same price. This "probability one" conclusion does indeed require $\varepsilon_{1t} \equiv 0$, which is another way of saying that $\tilde{p}(t,\infty)$ remains random (strictly positive variance) unless $\varepsilon_{1t} \equiv 0$. In contrast to the savings account, the permanent earnings dynamic depends on the idea of uncertain value-creation. And this condition of random ε_{1t} is the one that forces us to replace $p_t = x_{t+1}/r$ with $p_t = E_t[\tilde{x}_{t+1}]/r$.

Proposition I showed that $p_t = x_{t+1}/r$ suffices to characterize a savings account. In this spirit, one can ask: does the relation $p_t = E_t[\tilde{x}_{t+1}]/r$ combined with PVED suffice for the permanent earnings dynamic? The answer to the question is negative: PVED combined with $p_t = E_t[\tilde{x}_{t+1}]/r$ leads to neither $p_t = (R/r)x_t - d_t$ nor to the dynamic that defines permanent earnings.

A counter-example demonstrates the insufficiency of $p_t = E_t[\tilde{x}_{t+1}]/r$. It provides an opportunity to introduce the second benchmark earnings construct, namely economic earnings.[5] Economic earnings corresponds to a setting in which the market and book values coincide -- that is, it can be viewed as a perfect version of mark-to-market, or "fair value", accounting.

Assuming CSR, it follows that

$$x_t = \Delta p_t + d_t = \Delta b_t + d_t .$$

Economic earnings accordingly explain the *change* in value (adjusted for dividends) rather than value *itself*. Permanent and economic earnings thus constitute two perfect extremes by (fully) explaining either value or the change in value (though both cases require adjustments for dividends). They naturally complement each other as two baseline models.

Economic earnings as a construct would appear to be straightforward and essentially tautological. But the question arises how one characterizes the *dynamics* of (b_t, d_t) such that $b_t = p_t$ obtains as a conclusion rather as an a priori restriction. In other words, the proposition that follows must be consistent with Proposition II because the accounting dynamics defines the accounting construct.

Proposition III: Assume PVED. Then the economic earnings dynamic as defined by

$$\tilde{b}_{t+1} + \tilde{d}_{t+1} = R\left(b_t + d_t\right) - Rd_t + \tilde{\varepsilon}_{1t+1}$$

and the dividend dynamic

$$\tilde{d}_{t+1} = \theta_1\left(b_t + d_t\right) + \theta_2 d_t + \tilde{\varepsilon}_{2t+1}$$

combined with a no-excess-growth regularity condition imply

(i) $\quad p_t = b_t$

(ii) $\quad p_t = E_t\left[\tilde{b}_{t+1} + \tilde{d}_{t+1} - \tilde{b}_t\right] / r.$

If in addition CSR holds, then $\Delta b_t + d_t = x_t$ and (ii) reduces to

$$p_t = \frac{E_t\left[\tilde{x}_{t+1}\right]}{r}.$$

Comments:

- Like the permanent earnings setting, p_t's independence of θ_1 and θ_2 reflects DPI.
- To better appreciate the forecasting scheme, note that if one adds the CSR assumption. Then the economic earnings dynamic reduces to

  $$\tilde{x}_{t+1} = rb_t + \varepsilon_{1t+1}.$$

  The expression identifies a particularly simple forecasting scheme. Each dollar of book value forecasts *r* dollars of next-period earnings. Like permanent earnings, the forecast attribute is earnings. But the information used to forecast has changed from the flow variables x_t and d_t to the stock variable b_t. It is apparent that one needs to distinguish between the attribute forecasted and the information predicating the forecast.
- Assuming CSR, economic earnings is equivalent to residual earnings being unpredictable, $E_t\left[\tilde{x}^a_{1+\tau}\right] = 0$ Compare this to permanent earnings, which satisfies $E_t\left[\Delta\tilde{x}^a_{1+\tau}\right] = 0$.
- The parameterization of the book value dynamics is in terms of its cum-dividend book value, $b_t + d_t$ rather than b_t. Cum-dividend book value has the advantage of defining a variable that does not depend on the current dividend (similar to x_t which does not depend on d_t). Now the economic earnings dynamic is not all that different from the permanent earnings. The cum-dividend book value variable $b_t + d_t$ replaces x_t in permanent earnings, and the coefficient associated with d_t equals –*R* in lieu of –*r*. There are no other differences; Propositions II and III rely on the same derivations.[6]

Do the two earnings constructs have anything in common? Looking at the two valuation functions it may at first glance seem like they are poles apart. But a close look suggests otherwise.

Both permanent earnings and economic earnings satisfy DPI, and capitalized expected forward earnings determine value. These two intrinsic properties of the two benchmark models have been noted repeatedly. A more surprising similarity concerns the possibility of expressing the valuation function of economic earnings so it resembles the one associated with permanent earnings:

$$p_t = (R/r)E_{t-1}[\tilde{x}_t] - (d_t - \varepsilon_{1t}).$$

In this framework one thinks of $E_{t-1}[x_t] = rb_{t-1}$ as period-*t* earnings *before (unpredictable) windfall gains/losses.* The realized windfall gain/loss, ε_{1t} , has been combined with dividends, and it bypasses the measurement of ("dirty surplus") net income. A dollar of windfall loss has the same effect as a dollar of dividends on value. Using the jargon associated with GAAP, $x_t = E_{t-1}[\tilde{x}_t] + \varepsilon_{1t}$ where x_t equals comprehensive earnings, $E_{t-1}[\tilde{x}_t] = rb_{t-1}$ equals net income and ε_{1t} equals other comprehensive income (OCI to use the popular acronym). With this perfect mark-to-market framework the pre-OCI income measurement -- as opposed to comprehensive earnings -- is fully predictable for the forthcoming period. Thus by allowing for dirty surplus earnings the economic earnings model can be modified to make the valuation essentially the same as the one for permanent earnings.

The last point makes it clear that economic earnings and permanent earnings turn into identical models -- the savings account -- *if and only if there is certainty about the next period's earnings.* While the conclusion does not surprise in light of previous analysis, it underscores that the two sources of uncertainty, ε_{1t} and ε_{2t}, differ because the creation of wealth differs from its distribution in DPI settings. Again, only ε_{1t} is relevant.

Economic earnings differs from permanent earnings because the latter model does not require book and market to coincide. Yet the permanent earnings model provides an ideal construct of earnings, including the capitalization concept $p_t = E_t\left[\tilde{x}_{t+1}\right]/r$. How can earnings be ideal and yet there are missing, or improperly valued, assets/liabilities in the balance sheet? The next proposition sets the stage to answer this question.

Proposition IV: Assume PVED, CSR and that earnings satisfy permanent earnings. Define $g_t \equiv p_t - b_t$; then

$$E_t\left[\tilde{g}_{t+\tau}\right] = g_t \quad \text{all } \tau \geq 1.$$

Comments:

- From the perspective of economic earnings and a "fair value" balance sheet accounting approach, goodwill = g_t can be thought of as an "error" in the date *t* balance sheet. A change in goodwill accordingly reflects the "error" in earnings (assuming CSR). An accounting consistent with permanent earnings accordingly satisfies the following property: the expected error in earnings is zero. This in-expectation "canceling error" concept makes it plausible that the permanent earnings model, just like the economic earnings model, equates value to capitalized expected earnings.
- Proposition IV implies that the accounting is unbiased from a balance sheet perspective if one initializes such that $p_0 = b_0$. For $t \geq 1$, realized market and book values oscillate around each other with a trendless difference. On average, the accounting is neither

conservative nor aggressive.

- Proposition IV holds for economic earnings too. It is a trivial matter because $b_t = p_t$ implies $g_t = 0$ (which makes the expectation operator in $E_t\left[\tilde{g}_{t+\tau}\right] = 0$ redundant). The observation suggests that a focus on future expected outcomes, as opposed to the information used to forecast the outcomes, reveals the two models' common attributes. Note that $E_t\left[\tilde{x}_{t+\tau+1}\right] = RE_t\left[\tilde{x}_{t+\tau}\right] - rE_t\left[\tilde{d}_{t+\tau}\right]$ holds for both economic earnings and permanent earnings, given that τ is strictly greater than 0. For $\tau = 0$ the dynamic holds for permanent earnings but not for economic earnings. Permanent earnings and economic earnings therefore differ only in the information used to forecast. Putting aside this information issue, on sees that both benchmark models *satisfy permanent earnings in expectation*.
- One can take the last point one step further and claim that economic earnings is a confined version of permanent earnings. Specifically, given CSR, economic earnings corresponds to $E_t\left[\tilde{x}^a_{t+\tau}\right] = 0$ whereas permanent earnings corresponds to $E_t\left[\Delta\tilde{x}_{t+\tau}\right] = 0$ for all $\tau \geq 1$. The former condition implies the latter whereas the converse is false; in this sense permanent earnings is more general than economic earnings. This observation suggests, speaking in broad terms, that an earnings perspective provides additional leeway as compared to a book value perspective. If the latter satisfies what seems to be an appealing characteristic, then so does the former though the converse cannot be claimed. Subsequent sections revisit this idea.

The reader may reasonably ask if one can combine the two benchmark models (permanent earnings and economic earnings) to obtain the conclusion that p_t equals capitalized expected earnings. This cannot be done, in the following sense. Suppose we assume that (in addition to PVED) (i) $p_t = E_t\left[x_{t+1}\right]/r$, (ii) p_t must be a weighted average of the RHSs of the permanent earnings and economic earnings models. It can be shown that these assumptions imply that the solution is a corner case, that is either permanent earnings or economic earnings obtains; there can be nothing in between.[7] That said, there are other settings, consistent with DPI and PVED, that yield the conclusion $p_t = E_t\left[x_{t+1}\right]/r$. But these settings thus require an information set that goes beyond $\left(x_t, d_t, b_t\right)$. From this perspective the two benchmarks must be accorded a unique status.

Capitalized expected earnings serves as a benchmark for value. The question arises how one posits models that breaks the link and yet explains why p_0 and $E_t\left[\tilde{x}_{t+1}\right]/r$ differ. The ensuing sections address this question. How does one relate earnings and book values to price when neither permanent earnings nor economic earnings apply, except as special cases?

VI. Moving Beyond Capitalized Expected Earnings to Represent Value: Introducing Superior Growth

This section develops the concepts necessary to explain the price-to-forward earnings ratio in terms of growth. The broad issue is straightforward: we need to go beyond the starting point 1/*r* and to do so we have to elaborate on growth. As prior analysis has suggested, the idea is to articulate the nature of superior growth and to show how it increases value given any forward earnings and discount factor. Investment practice, of course, aligns with such thinking. As to theory, the primary analytical objective is to clarify the precise meanings of growth, or equivalently, why the price-to-forward earnings ratio differs from 1/*r*. There can be alternative measures of growth, and each has its particular influence on the price-to-forward earnings ratio.

Three points will be stressed in this section. First, whether one considers growth in expected earnings or book values, such growth depends on dividends and the retention of earnings. Previous section noted that the greater the retention the greater the growth, ceteris paribus. In light of this we conceptualize superior growth by showing how one makes the appropriate adjustment due to dividends when one measures growth. Thus the proper growth measures will be dividend policy neutral. Second, the analysis considers not only the growth in earnings but also the growth in book values (both adjusted for dividends) to explain the price-to-forward earnings ratio. The third point I stress is that it makes sense to think about the growth-in-book-values approach as a special case of the growth-in-earnings approach: modeling of the former can be transformed into the latter, but the converse is not generally valid.

In contrast to the previous section, the analysis makes no reference to some underlying dynamic spelling out the information used to forecast future outcomes. Sequences of expected outcomes are exogenous, and the analysis addresses how these future expected outcomes convert into today's value. Approaching the price-to-forward earnings ratio question from this more limiting perspective provides the first step. Following this analysis, the next section identifies supporting information dynamics. It will become apparent how the dynamics of the permanent earnings and economic earnings generalize to allow for earnings growth in addition to the effect due to retained earnings.

To simplify the mathematical expressions, from now on in this section date 0 specifies the valuation date and the notation suppresses the expectation operator (which also means that the date 0 information is unspecified).

A mathematical equality provides the analytical starting point: For any sequence of numbers $y_0, y_1, ...$

$$0 = y_0 + R^{-1}\left(y_1 - Ry_0\right) + R^{-2}\left(y_2 - Ry_1\right) + ... = y_0 + \sum_{t=1}^{\infty} R^{-t}\left(y_t - Ry_{t-1}\right),$$

provided that $y_t / R^t \to 0$ as $t \to \infty$

Adding PVED and the (convergent) zero-sum series yields

$$p_0 = y_0 + \sum_{t=1}^{\infty} R^{-t} z_t,$$

where

$$z_t \equiv y_t + d_t - Ry_{t-1}.$$

To be sure, the analysis works for any *y*-sequence, and *z*-sequence, as long as the transversality condition is met. The idea is that one can represent value in terms of two parts: a starting point, y_0, and a complement defined by the present value of a generic sequence, $z_1, z_2, ...$ which embeds the sequence of dividends. Though the relations are trite, they speed up and streamline derivations of subsequent insights.

The reader may already have noticed that RIV follows by putting $y_t = b_t$ and combining it with CSR. In a similar spirit, the Abnormal Earnings Growth (AEG) model follows if one puts. $y_t = x_{t+1} / r$.[8]

RIV depends on CSR, as is well-known. But in this context CSR obscures that the essence of RIV refers to $y_t = b_t$, which has nothing to do with earnings or CSR per se. By leaving out the CSR step, one obtains the Abnormal Book Value Growth (ABG) representation. Focusing on ABG rather than RIV makes it clear that ABG and AEG differ only because either b_t or, alternatively, x_{t+1} / r specifies y_t. For ABG $z_t = b_t + d_t - Rb_{t-1}$ whereas for AEG

$z_t = x_{t+1} / r + d_t - Rx_t / r$.[9]

Contrary to what the literature often suggests, RIV/ABG and AEG should not really be labeled as "models" of value. It misleads insofar that the acronyms, without further assumptions or restrictions, lack the richness required to make the case that (forecasted) earnings or book values are intrinsically useful in a valuation context. Though the upbeat nature of the claim appeals, it must be tempered by the observation that the derivations do not depend on any conceptual restrictions on accounting data. There is, for example, no clear distinction between the distribution and creation of wealth. It is better to think of these acronyms as stage-setting, mechanically equivalent, representations of PVED. They do not depend on any assumptions per se, and their usefulness hinge on the possibility of introducing assumptions on the purported accounting data such that the variables represented have either empirical or conceptual content. Only under such conditions will the variables x and b "deserve" their labels (and of course such assumptions have to go beyond the CSR restriction).

Permanent earnings and economic earnings illustrate how additional assumptions imply sharp conclusions. For permanent earnings, put $y_t = x_{t+1} / r$ so that $z_t \equiv (x_{t+1} + rd_t - Rx_t) / r = 0$ and hence via AEG $p_0 = x_1 / r$ since PV of z = 0. For economic earnings, put $y_t = b_t$ so that $z_t \equiv b_t + d_t - Rd_{t-1} = 0$ and hence via ABG $p_0 = b_0$ since, again, PV of $z = 0$.

Neither RIV/ABG nor AEG requires the z_t to equal zero, of course. Only benchmark models by their nature correspond to the case $p_0 = y_0$; the second term in the representations RIV/ABG or AEG, the PV of the z-sequence, equals zero simply because all elements in this sequence equal zero. Moving away from benchmark models, it becomes apparent that, more generally, cases when the z_t exceed zero represent positive growth, which results in a premium, $p_0 - y_0 > 0$. This analysis applies whether y_0 is x_1 / r or b_0. Moreover, this growth is superior in the sense that it captures growth that goes beyond retained earnings. In other words $y_t + d_t - Ry_{t-1}$ is, in fact, a dividend adjusted growth. Now the question arises if one can find some useful, additional assumption that parameterizes this superior growth.

VII. Parameterized Models Explaining the Price-to-Forward Earnings Ratio

In what follows I do not attach any interpretation to the y -sequence. Before exploiting the two natural choices, earnings (capitalized) and book values, a useful mathematical result will be stated. *One can derive* p_0 *as a function of* y_0 *and* $y_1 + d_1$ *alone, given suitable assumptions on the* y *-sequence, or, equivalently, the* z *-sequence*. Unsurprisingly, the assumption requires the *z*-sequence to grow (or decay) geometrically.

Proposition V: Assume PVED. Consider any sequence $\{y_t\}_0^\infty$ satisfying $\lim_{t\to\infty} y_t / R^t = 0$ and a related sequence $z_t \equiv y_t + d_t - Ry_{t-1}$ such that

$$z_{t+1} = \gamma z_1,\ \ t \geq 1,\ 0 < \gamma < R.$$

Then

$$p_0 = y_0 + \frac{z_1}{R-\gamma} = y_0 \cdot \frac{\left[\dfrac{y_t + d_1}{y_0} - \gamma\right]}{(R-\gamma)}$$

and

$$p_0 = wy_0 + (1-w)\frac{y_t + d_1}{R}$$

where

$$w = -\frac{\gamma}{(R-\gamma)} < 0.$$

Armed with these analytical results, I next identify the two cases that explain the price-to-forward earnings ratio. The first approach, (i), refers to the growth in book value, and the second, (ii), refers to the growth in earnings.

(i) Put $y_t = b_t$. This setting starts from ABG. An assumption of geometric growth (decay) in $z_t = b_t + d_t - Rb_{t-1}$ simply changes the notation and the last proposition reduces to

$$p_0 = b_0 \cdot \left[\frac{(b_1 + d_1)/b_0 - \gamma}{R - \gamma} \right].$$

One reads $(b_1 + d_1)/ b_0$ as the forthcoming growth in the expected book value, adjusted for dividends. The numerator adjustment for dividends is crucial: it reflects DPI. That is, the numerator does *not* depend on the next period's dividend decision since the *cum*-dividend book value, $b_1 + d_1$, does not depend on the dividend. Thus the choice of date-one dividends does not affect p_0.

Standard derivations of the model assume CSR. It leads to the textbook expression:

$$p_0 = b_0 \cdot \left[\frac{roe_1 - (\gamma - 1)}{r - (\gamma - 1)} \right],$$

where roe_1 equals the forthcoming expected return on equity, $roe_1 \equiv x_1 / b_0$. It follows that p_0 / b_0 increases as roe_1 increases. With respect to γ -- where now $x^a_{t+1} = \gamma x^a_t$ -- increases as γ increases, when $roe_1 > r\left(roe_1 = r \text{ renders } \gamma \text{ irrelevant}\right)$. These conclusions make sense since they reduce to the idea that "growth in residual earnings is good assuming they are initially positive."

Relating the market-to-book ratio to roe_1 has some appeal, of course. But it lacks a compelling real-world motivation. Investors tend to ask: "what factors explain the price-to-forward earnings ratio?" rather than "what factors explain the market-to-book ratio?" More important for our purposes, the last question is of interest since it has already been established that $p_0 = x_1 / r$ holds no less for economic earnings than for permanent earnings. A model resting on book values ought not to rule out an explanation of p_0 / x_1.

Shifting the focus to the price-to-forward earnings ratio, simple manipulations of the last equation leads to

$$p_0/ x_1 = K_1 + K_2/ roe_1$$

where

$$K_1 = 1/(R - \gamma)$$

$$K_2 = (1 - \gamma) /(R - \gamma).$$

The LHS variable of interest, the price-to-forward earnings ratio, depends only on the RHS variable roe_1, aside from the parameters γ and R.

An evaluation of how roe_1 influences p_0/ x_1 depends on the sign of K_2. Signing K_2 in turn puts the onus on the sign of $1-\gamma$. Is γ greater or less than one? It makes sense to require γ to exceed 1 if x_t^a is positive (i.e., $roe_1 > r$), just as γ should be less than one if x_t^a is negative $\left(roe_1 < r\right)$. The first claim is based on the idea that if the firm is profitable, then, in expectation, the dollar amount of goodwill (or residual earnings) should expand with time. Such an expected scenario occurs if conservative accounting is combined with (expected) growth in the business (If $roe_1 > r$ but $\gamma < 1$, then x_t^a and $p_t - b_t$ decline with t and both go to zero, which is inconsistent with conservative accounting (in expectation)). As the second possibility, if the firm is unprofitable $\left(roe_1 < r\right)$, then one should expect that the gap, roe_1 vs. r, to be gradually closed in the future. And given $roe_t < r\left(\text{or } x_1^a < 0\right)$, x_t^a goes to zero as $t \rightarrow \infty$ if and only if $\gamma < 1$. Thus $roe_1 < r$ leads to the condition $\gamma < 1$.

Given the above restrictions -- summarized by $x_1^a(\gamma - 1) > 0$ -- it follows that if roe_1 is less than r , then the price-to-forward earnings ratio, p_0 / x_1, *de*creases as roe_1 increases. For roe_1 greater than r , the price-to-forward earnings ratio now increases as roe_1 increases. In other words, as an empirical matter one should expect the function p_0 / x_1 on roe_1 to be U-shaped.

Though the analysis may seem somewhat turgid and mechanical, it makes more intuitive sense than one might think initially. Consider a firm with roe_1 of say 14% when r = 10%. Such a firm is profitable, and the setting corresponds to $\gamma > 1$. Now it is clear that if the firm remains about equally profitable in the future, then a back-of-the-napkin calculation shows that the growth in expected earnings will be superior. Hence the price-to-forward earnings ratio ought to show a premium (exceed 1/*r*). Next consider when roe_1 is poor, say 2%, which is the setting when $\gamma < 1$. Now there is reason to expect that *roe* improves with the passage of time. A back-of-the-napkin calculation now shows that even a modest improvement in *roe*, to say 3%, implies a considerable growth in expected earnings (FY2 vs. FY1). Again, the growth principle leads to the conclusion that the price-to-forward earnings ratio reflects a premium. (As an empirical matter, it is easily verified, looking at real-world data, that the great majority of firms with sub-par roe_1, such as less than 5%, indeed have relatively large price-to-forward earnings ratios. Yet, as any text-book will note, such firms will also have below average market-to-book ratios.)

The modeling allows for the case when the capitalized forward earnings alone determine value: $\gamma = 1$ or $roe_1 = r$ provide the necessary and sufficient conditions. The case $roe_1 = r$ is rather trite since now $x_t^a = 0, p_0 = b_0$ and $b_0 = x_1 / r$. The idea behind $\gamma = 1$ is more subtle; now the conclusion follows even though $p_0 - b_0 \neq 0$ (the sign depends on the sign of x_t^a, of course).

However, $\gamma = 1$ implies $p_0 - b_0 = p_1 - b_1$ and hence the expected balance-sheet valuation "error" in the next year cancels with the one in the current year. And such cancelling of error (in expectation) leads to a perfect measure of expected earnings so that $p_0 = x_1 / r$. The absence of growth is the other side of the coin.

Profitability, as a concept, refers to the dividend-adjusted growth in *book value*, and $(b_1 + d_1)/b_0$ explains p_0 / x_1 whether one assumes CSR or not. Again, CSR obscures rather than helps. To further bring out that the model centers around book values, note that the proposition implies

$$p_0 = wb_0 + (1-w)\frac{b_1 + d_1}{R}$$

where $w = -\gamma/(R - \gamma) < 0$. Referring back to Proposition III's benchmark model for book values without CSR, w will be irrelevant because under the assumptions of Proposition III, $p_0 = b_0 = (b_1 + d_1)/R$. This setting thereby eliminates any growth except what is implied by retained earnings. Hence the more general model admits for a *superior* growth in book values, $((b_1 + d_1)/b_0 - 1) > r$, or $b_1 + d_1 > Rb_0$, to explain a price-to-forward earnings ratio different from $1/r$. And to appreciate this growth construct, note that p_0 increases as b_0 decreases, holding $b_1 + d_1$ fixed.

(ii) Put $y_t = x_{t+1}/r$. This setting meshes with the AEG framework. A geometric growth (decay) assumption now leads to the OJ valuation formula

$$p_0 = \frac{x_1}{r} \cdot \left[\frac{g_2 - (\gamma - 1)}{r - (\gamma - 1)}\right],$$

where

$$g_2 \equiv \frac{\Delta x_2 + rd_1}{x_1}$$

defines the short-term growth in expected earnings, adjusted for dividends. Like the book value model, DPI builds in an adjustment for dividends. In the present case the idea is that $x_{t+2} + rd_t$ does *not* depend on dividends because x_2 depends directly on d_1. A savings account illuminates the concept because it shows that d_1 foregoes earnings in the subsequent period such that $x_{t+2} + rd_1$ is independent of d_1.

Ohlson and Jeuttner (2005) note that under mild assumptions on the dividend policy γ equals the long term growth in expected earnings:

$$\lim_{t\to\infty}\left(x_{t+1} / x_t\right) = \gamma$$

Hence the OJ model subsumes the idea that the short- and long-term growth (STG and LTG) in expected earnings determine the price-to-forward earnings ratio.

To evaluate the function that explains the price-to-forward earnings ratio as a function of the growth measures,

$$p_0 / x_1 = k_1 + k_2 g_2,$$

where

$$k_1 = -(\gamma - 1)/(r(R-\gamma))$$

$$k_2 = 1/(r(R-\gamma)) > 0.$$

Hence p_0 / x_1 increases as STG increases, as ought to be the case. With respect to γ, the dependent variable increases in γ if one presumes g_2 exceeds the benchmark r. It, too, makes sense since γ represents LTG. For $g_2 = r$, LTG is now irrelevant since the current growth is neutral. If $g_2 < r$, a case of inferior growth, then it makes sense to assume that γ is less than 1. The reasoning for this parameterization is of course the same as in the case when $y_t = b_t$. Now

p_0 / x_1 decreases as γ increases. But this restriction does not affect whether p_0 / x_1 increases as STG increases (given fixed γ and r): it always increases as g_2 increases.

The special case when capitalized forward earnings suffices to determine value occurs if and only if $g_2 = r$ (equivalently, $z_1 = 0$). The pricing reflects no premium unless there is superior growth in expected earnings.

Finally, to underscore the earnings growth perspective, express value as

$$p_0 = w\frac{x_1}{r} + (1-w)\frac{x_2 + rd_1}{rR},$$

where $w = -\gamma /(R-\gamma) < 0$. Similar to the previous setting (when $y_t = b_t$), the weight is possibly irrelevant. This occurs if and only if $p_0 = x_1 / r = (x_2 + rd_1)/ rR$. The last relations hold for the permanent earnings model and a savings account.[10,11] Relaxing this condition, $w < 0$ leads to the pronounced effect of growth in expected earnings: p_0 increases as x_1 decreases if one keeps $x_2 + rd_1$ fixed.

Do the two models $y_t = b_t$ and $y_t = x_{t+1} / r$ provide alternative, competing, approaches to valuation? As a theoretical matter the answer is "no". Given CSR, it turns out that *the M-to-B model is a special case of the OJ model*. To see why this must be true, first note that the OJ model with CSR implies that $z_t = \frac{1}{r}\Delta x_t^a$. It follows that the model satisfies $\Delta x_t^a = \gamma\Delta x_{t-1}^a$, which compares to the dynamics of the M-to-B model, $x_t^a = \gamma x_{t-1}^a$. Second, the M-to-B dynamic implies the OJ dynamic, but the converse is false (consider the dynamic $x_t^a = \gamma x_{t-1}^a + K$, which implies $\Delta x_t^a = \gamma\Delta x_{t-1}^a$ even if $K \neq 0$).

Proposition VI: The assumptions that imply the M-to-B model (i.e., PVED; CSR; $x_{t+1}^{a} = \gamma x_{t}^{a}$, for $t \geq 1$) also imply the OJ model. The converse is false.

Simple as the analysis is, one arrives at the following conclusion: the idea that the growth in earnings, STG and LTG, explains the price-to-forward earnings ratio is a principle of generality.[12]

VIII. The Sustaining Information Dynamics

This section states the two information dynamics that support the parameterized RIV and AEG models, in other words the M/B-model and the OJ model.[13] Both of the dynamics should be thought of as generalizations of, respectively, economic earnings and permanent earnings. A full appreciation of the current section accordingly depends on an understanding of the previous discussion of the two benchmark models. Hence the modeling extends the benchmarks by superior growth. This generality implies that the price-to-forward earnings ratio exceeds $1/r$. Simplicity and symmetry will be part of the modeling: having considered one of the dynamics it becomes more or less automatic what the second must be. Each of the two dynamic have two ingredients: (i) a starting point as determined by either economic earnings or permanent earnings, (ii) information that bears on the forthcoming growth. With those ideas in place one can next augment the dynamic via period-idiosyncratic information which makes the empirical content of the model more realistic. Finally, I show the ways in which the modeling articulates accounting conservatism.

The first case extends the economic earnings benchmark such that it leads to the parameterized RIV model and the M-to-B model. Consider the dynamic equations

$$\begin{aligned}\tilde{x}_{t+1} &= rb_t + \nu_{1t} + \tilde{\varepsilon}_{1t+1} \\ \tilde{\nu}_{1t+1} &= \gamma\nu_{1t} + \tilde{\varepsilon}_{2t+1}\end{aligned} \qquad \text{[ID-1]}$$

where the disturbance terms have zero mean in the usual fashion. A dividend policy equation is understood, but, because DPI applies, its dynamic specification can be skipped. With respect to the incremental notation, one thinks of ν_{1t} as generic "other" information.[14] It can depend on virtually anything inside and outside the financial statements.

The second case generalizes permanent earnings that lead to the OJ model through the AEG framework. Consider the dynamic equations

$$\begin{aligned}\tilde{x}_{t+1} &= Rx_t - rd_t + \nu_{1t} + \tilde{\varepsilon}_{1t+1} \\ \tilde{\nu}_{1t+1} &= \gamma\nu_{1t} + \tilde{\varepsilon}_{2t+1}\end{aligned} \qquad \text{[ID-2]}$$

with the standard assumptions on the disturbance terms. (The comments following ID-1 apply no less for ID-2.)

Each of the two dynamics implies a valuation solution which expresses how the date t price depends on the date t information and the parameters r, γ.

Proposition VII: Assume PVED and consider two information dynamics.

For ID-1 value equals

$$p_t = b_t + \frac{1}{(R-\gamma)}\nu_{1t},$$

For ID-2 value equals

$$p_t = \frac{R}{r}x_t - d_t + \frac{R}{r(R-\gamma)}\nu_{1t}.$$

Moreover,

$$p_t = E_t[\tilde{x}_{t+1}]/r + q\nu_{1t}$$

where $q = 1/(R-\gamma) - 1/r$ (ID-1) and $q = R/r(R-\gamma) - 1/r$ (ID-2), respectively.

It is apparent that ID-1 implies the M/B-model, which in turn implies the OJ model. ID-2, on the other hand, implies only the OJ model. In this sense ID-2 provides a more robust valuation setting. That said, for both models one readily explains the sign of $p_t / E_t[\tilde{x}_{t+1}] - 1/r$ via a growth construct.

Corollary: For both ID-1' and ID-2',

$$p_t / E_t[\tilde{x}_{t+1}] > 1/r$$

if and only if there is growth in the (dividend-adjusted) expected earnings

$$E_t[\Delta\tilde{x}_{t+\tau} + r(\tilde{x}_{t+\tau} - \tilde{d}_{t+\tau})] > 0 \text{, for all } \tau \geq 1.$$

The above result usefully brings out that one can think of the premium in the price-to-forward earnings ratio and growth in expected earnings as two sides of the same coin: each implies the other.[15] The result appeals in its robustness -- it works for a book value perspective as well as one on earnings -- and it aligns with a principle held to be a truism in investment practice.

The assumptions on the dynamics confine the analysis insofar that they only add one piece of information, ν_{1t}. And this piece of information uniquely augments the forecasts of next period's expected earnings as well as the (dividend adjusted) growth. As a generalization of the ID's one may consider adding some period-idiosyncratic information that affects the forecasting of $E_t[\tilde{x}_{t+1}]$ but has no influence of the underlying growth process (the second dynamic equation remains the same). Hence, consider ID-1' and ID-2':

$$\tilde{x}_{t+1} = rb_t + \nu_{1t} + \nu_{2t} + \tilde{\varepsilon}_{1t+1} \qquad \text{[ID-1']}$$

and

$$\tilde{x}_{t+1} = Rx_t - rd_t + \nu_{1t} + \nu_{2t} + \tilde{\varepsilon}_{1t+1}, \qquad \text{[ID-2']}$$

where $E_{t-1}[\tilde{v}_{2t}] = 0$.

ID-2' supports the OJ model. The idiosyncratic information has no effect on the formula because x_t, d_t, v_{2t} are of no interest given $E_t[x_{t+1}]$, and this variable (scaled by $1/r$) provides the starting point in the OJ formula. Referring back to Proposition VI, v_{1t} is proportional to z_t for all $t \geq 1$ ($t = 0$ is irrelevant) and thus all conditions of that proposition are met. As to ID-1', it, too, supports the OJ model for the same kind of reasoning. But the M/B model does not hold because the idiosyncratic information prevents the current book value to act as the sole starting point in the valuation. It is as if v_{2t} garbles the information inherent in the current book value. That said, the idiosyncratic information is unforecastable for the next period which means that the M/B model holds on average; it is expected to hold at the next date: $E_t[\tilde{p}_{t+1}] = E_t[\tilde{b}_{t+1}] + \dfrac{E_t[\tilde{x}^a_{t+2}]}{R-\gamma}$. From all of this one learns that it in the presence of idiosyncratic information it makes a difference whether the date t valuation starts from a variable with a subscript t+1 -- like $E_t[x_{t+1}]/r$ -- or t -- like b_t.

Idiosyncratic information is of interest because it makes the modeling less confined as an empirical matter. Without it the dynamics restrict ID-1' so that p_t - b_t is positive and for ID-2' p_t always exceeds $(R/r)x_t$ - d_t. The added degree of freedom removes these unrealistic features. For both dynamics, one can now obtain any configuration of signs for p_t - b_t and p_t - $((R/r)x_t$ - $d_t)$, including (-,+) and (+,-).

Though idiosyncratic information invalidates Proposition VI, one can modify the conclusions there by shifting the focus from current valuation to the expected future valuation. As noted, no problem arises because of the unpredictability of idiosyncratic information. For both of the dynamics it follows immediately that the expected future price at date t depends only on the contemporaneous expected book values and expected earnings/dividends , for ID-1' and

ID-2' respectively, and the expected value of v_{1t} also at the future date t. Given a growth scenario $\gamma > 1$ and $v_{1t} > 0$, it follows that for all future dates the expected forward earnings capitalized will be less than the expected price, and the claim holds for both dynamics. There is no surprise here in light of the discussion that followed Proposition VI. Less obvious, for ID-2' as well as for ID-1' it also follows that asymptotically the expected price will exceed the expected book value. From a theory perspective, the result appeals because it shows that a straightforward modification of either of the two benchmark models, permanent and economic earnings, which introduces growth and idiosyncratic information, leads to conservative accounting in the balance sheet and the income statement whenever there is growth.

Proposition VIII: Assume PVED, and either ID-1' or ID-2' with $\gamma > 1$. Then

$$E_t[\tilde{p}_{t+\tau} - \frac{\tilde{x}_{t+\tau+1}}{r}] > K > 0, \text{ for any } \tau = 0, 1, \ldots.$$

And given CSR,

$$\lim_{\tau \to \infty} E_t[\tilde{p}_{t+\tau} - \tilde{b}_{t+\tau}] > K > 0 .$$

A proof of the balance sheet conservatism in expectation for ID-2' exploits that $E_t[\tilde{p}_{t+\tau} - \tilde{b}_{t+\tau}]$ does not depend on the dividend policy because of DPI. Hence one can put $d_t = x_t$ so that $b_{t+\tau} = b_t$ for all τ. The condition now follows because $E_t[\tilde{p}_{t+\tau} - \tilde{x}_{t+\tau+1}/r] > 0$ and $E_t[\tilde{x}_{t+\tau}/r]$ increases geometrically as $\tau \to \infty$ when $\gamma \geq 1$.

The last proposition looks mechanical in its exposition of conservatism. But it can be aligned with basic economics and accounting. Feltham and Ohlson (1995) suggest conservatism arises due to two distinct sources. First, there exist positive NPV projects. Second, the accounting rules can be intrinsically conservative.

As to the first point, a firm may be perceived to have the opportunity, in expectation, to undertake strictly positive NPV projects. Such projects do not affect the accounting today, but the same is not true for today's value. Hence, it goes almost without saying that the market value today can exceed both book value and next periods expected earnings capitalized. (A general analysis exploits RIV and AEG). Nevertheless, the last two propositions makes it clear that this source of conservatism can occur only in the context of growth. Stated somewhat differently, neither permanent earnings nor economic earnings can co-exist with a positive NPV environment unless, unnaturally, the opportunities are "booked" when perceived rather than as a consequence of execution and outcomes. It is seen that the introduction of growth predicates any introduction of positive NPV projects.

It may seem that positive NPV (in expectation) is not only sufficient but also necessary for the conservatism conclusion. Such is not the case, however. Feltham and Ohlson's provide a second point: the accounting rules themselves can be conservative quite aside from not recognizing positive NPV opportunities. Accelerated depreciation, R&D accounting, excess write-offs illustrate. Thus one can readily obtain an inequality where price exceeds book value in expectation (as well as in realization) solely due to a downward bias in the balance sheet valuation rules. If in addition there is business growth then earnings will also be relatively depressed because they will be effectively "deferred". Of course, the modeling in this section has not distinguished between the two kinds of conservatism sources: Proposition VIII allows for both types of conservatism.

IX. Comments on Ohlson (1995) and Feltham and Ohlson (1995)

At no point has this paper made an explicit or implicit reference to the dynamics in Ohlson (1995), O95 henceforth, and Feltham and Ohlson (1995), FO95 henceforth. This omission is quite deliberate, which is not to say that many of the underlying ideas in these two papers have not resurfaced in the present one. Indeed, any one familiar with these two papers will recognize that concepts such as DPI, RIV, and permanent and economic earnings show up in these papers. That said, the omission leads to the question: what are the "deficiencies" of the O95 and FO95 dynamics? Without going into any details concerning these dynamics, the remainder of this section discusses this question.

Put succinctly, the aforementioned papers violate some of the core principles stated in the previous section. Turning to O95 first, the dynamic in this paper is a weighted average of permanent and economic earnings plus "other information." This approach leads to unbiased accounting in capitalized earnings and book values, that is, the latter two value indicators on average equal price. In this way the two bottom lines have been accorded similar status in the analysis. While the model may be of some theoretical interest, on aesthetic grounds two objections immediately spring to mind.

As the first objection, accounting theory has long recognized that the "the equivalent status" concept of book values and earnings makes little sense if on thinks of the consecutive balance sheets, and related book values, as a means to an end, namely the measurement of earnings (via CSR). Consistent with the discussion in this paper, the means-to-an-end perspective on the balance sheet exploits the cancelling error concept which in turn leads to capitalized forward earnings as a starting point in valuation. The valuation model in O95 does not, and can not, exploit the cancelling error concept to assign a privileged role for earnings and capitalized forward earnings as a useful starting point in valuation. In other words, the O95 dynamics run counter to a foundation that leads up to Proposition VIII and its implication that (dividend-adjusted) growth in earnings explains the price-to-forward earnings ratio. As the second objection, O95's unbiased accounting by definition rules out conservative accounting, which is no doubt more descriptive empirically. It follows that the O95 model also rules out robust long-term growth in expected earnings beyond retained earnings, again casting doubt on its empirical and practical usefulness.

FO95 reflects somewhat different problems though there is some "spillover" from O95. It does allow for conservatism and growth. But it does so in an ad hoc fashion insofar that it commingles "other information" with "net operating assets." Thus the paper does not make it clear whether the conservatism/growth rests on "other information" or "net operating assets" or both. FO95 also misses out on the idea that the dynamics could have used economic earnings rather than permanent earnings as its foundation (in the absence of "other information" and "net operating assets"). One can argue, therefore, that FO95 is best thought of as an exercise which exploits a somewhat arbitrary model to show how conservatism and growth can manifest themselves. It contrasts with the previous sections in this paper: it proceeds in a systematic fashion to show the intertwined nature of growth and conservatism, regardless whether permanent or economic earnings serve as the underlying benchmark model.

X. Concluding Remarks

Researchers who try to articulate theory concepts must deal with the fact that any application of the rules of logic (the process of moving from assumptions to conclusions) has nothing to do with labels attached to the symbols introduced and manipulated. The present paper is no exception, of course. Thus the question arises if there are valid reasons for the labeling used. In the present context the reader has to ask if x and b "deserve" the labels of book values and earnings, respectively. Why not something else, like cash balance or cash flows? (Most readers, I presume, are comfortable enough with d as dividends.) Absent a reasonable justification for the labeling of the abstract symbols the reader might well view the logical exercises as being of merely academic interest, and where the word "academic" suggests that the practical and empirical implications are moot (at best). In the next paragraph I argue that the analysis has provided a conceptually useful foundation for the study of earnings, book values, and dividends as to how these variables relate to equity value. In the paragraph thereafter the discussion makes the case that the analysis is also of practical interest.

Why does the analysis appeal? In broad terms, because many of the expressions that relate value to accounting data capture concepts inherent in accounting. Thus the x-variable can be labeled earnings for reason other than “I, as a researcher, postulate it to be so”. Accounting, or the financial reporting model, has its own imperatives, and these make their presence felt throughout. The CSR takes on a role when one distinguishes between the book values and earnings. Book values differ from earnings because the latter requires a capitalization factor to be of the same order of magnitude as book values. With respect to d, the analysis distinguishes between the creation vs. the distribution of wealth: the d does indeed differ from the x and b, and the shift from PVED to various expressions that incorporate the x and the b embeds the simple but important (“MM”) idea that the distribution of value must reconcile and be consistent with its creation. Wealth creation takes on prominence insofar that the dividend policy itself cannot create any value, i.e., DPI applies. Implementation of this concept depends critically -- and appealingly -- on the three accounting concepts: (i) dividends do not influence same date earnings; (ii) dividends reduce book value and (iii) dividends reduce subsequent (expected) earnings since the reduced book value represent less resources essential to generate future earnings. Only with these properties in place do the dividend neutral constructs $b_t + d_t$ and $x_t + rd_{t-1}$ come into sharp focus as representations of value creation. Moreover, given CSR, the analysis brings out the role of "canceling error" in consecutive (expected) book values. In turn, this concept results in earnings being the central valuation attribute. The driving idea here is compelling: "good" balance sheets lead to "good" income statements, but the converse is not true.

Does the analysis result in useful practical implications? I think so for the simple reason that investment practice starts from the principle that the growth of expected earnings should justify the price-to-forward earnings ratio. And this is the principle that the analysis elaborates on, including "why earnings" and the nature of the growth constructs. It certainly improves on the traditional, textbook, so-called constant growth model (often attributed to Gordon or Williams). The popularity of this model has been more dependent on the importance of the practical issues that it can address rather than its intrinsic appeal. The OJ model can be put to use in a similar fashion to deal with the practical issue of estimating cost of capital as well as estimating a firm's intrinsic value itself. It does so without an overhang of ambiguous parameters lacking in specificity or how they should be interpreted. The OJ model also permits various extensions such as how it can be refined to handle settings distinguishing between operating versus financial activities, and it reconciles with standard models that rest on taking the PV of free cash flows. Ohlson and Gao (2006) discuss these extensions in some detail. Thus one can reasonably claim that no matter what a more sophisticated yet practically useful theory of valuation will look like, it is unlikely that such a theory will be independent of and irreconcilable with the basic analytics that this paper articulates.

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ENDNOTES

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[1] References in Christensen and Feltham (2003), Chapter 9 and 10, list most papers that deal with accounting data and value up to 2003. Most (relevant) papers published subsequent to 2003 show up as references in this paper. Also, see Richardson and Tinaikar (2004) for a recent broad review of accounting data and value models.

[2] An arbitrary (and peculiar) policy like $d_t = 0$, t=1,..., 100 and $d_t = \sin^2(x_t)x_t$ for t > 100 works no less than a fixed payout. The payout function $k_t = \sin^2(x_t)$ satisfies the property that $k_t \in (0, 1)$ yet there is in no sense a meaningful pattern as t = 1,2,…. That is, k_t will be effectively random though it is technically deterministic. And it guarantees $\lim_{t\to\infty} x_t / R^t = 0$, which is the necessary and sufficient regularity condition on the payout policy.

[3] Permanent earnings, as an analytical construct, appears to be due to Ryan (1986). Other individuals, such as Fisher Black and Bill Beaver, have written on the idea that earnings "ought to" serve as an indicator of value.

[4] The Permanent earnings construct can be modeled as a function of cash flows and other valuation relevant events. See Ohlson and Zhang [1999].

[5] Many individuals refer to economic earnings as Hicks' concept of earnings.

[6] To see the analytical equivalence more clearly, consider the following dynamics:

$$\begin{aligned}\tilde{h}_{t+1} &= Rh_t - d_t + \tilde{\varepsilon}_{1,t+1}\\ \tilde{d}_{t+1} &= \theta_1 h_t + \theta_2 d_t + \tilde{\varepsilon}_{2,t+1}\end{aligned}.$$

with the usual assumptions concerning the disturbance terms and the regularity condition on the matrix $\begin{bmatrix} R & -1 \\ \theta_1 & \theta_2 \end{bmatrix}$. Combined with PVED, one readily shows that

$$p_t = Rh_t - d_t.$$

permanent earnings accordingly correspond to $h_t \equiv x_t/r$, and economic earnings correspond to $h_t \equiv (b_t + d_t)/R$.

[7] A slightly more general result can be stated. Assume: (i) PVED, (ii) DPI, (iii) $p_t = w_1(\frac{R}{r}x_t - d_t) + w_2 b_t$, and (iv) $p_t = E_t[\tilde{x}_{t+1}]/r$. Then (w_1, w_2) equals either (0, 1), or (1, 0). Thus there are only two benchmarks. The result follows from Ohlson (1995), page 676.

[8] One generalizes the Gordon-Williams divided growth setting by putting $y_t = d_{t+1}/r$. With this specification $z_t = \frac{d_{t+1}}{r} + d_t - R\frac{d_t}{r} = \frac{\Delta d_{t+1}}{r}$, and $p_0 = \frac{d_1}{r} + \sum_{t=1}^{\infty} R^{-t}\frac{\Delta d_{t+1}}{r}$: the yield premium $p_0 - d_1/r$ is explained by the subsequent growth in expected dividends.

[9] Assuming CSR, AEG derives from RIV via two simple steps. First, $p_1 + d_1 - p_0 = b_1 + d_1 + \sum_{\tau=1}^{\infty} R^{-\tau} x_{\tau+1}^a - (b_0 + \sum_{\tau=1}^{\infty} R^{-\tau} x_\tau^a) = x_1 + \sum_{\tau=1}^{\infty} R^{-\tau} \Delta x_{\tau+1}^a$. Second, PVED is equivalent to $p_1 + d_1 - p_0 = rp_0$. Combining the two expressions results in $p_0 = \frac{1}{r}(x_1 + \sum_{\tau=1}^{\infty} R^{-\tau} \Delta x_{\tau+1}^a)$, which defines AEG if CSR applies.

[10] The parameterization $\gamma = 1$ (or $w = -1/r$) is of interest. For $y_t = b_t$ it was noted that $p_0 = (b_1 + d_1 - b_0)/r = x_1/r$. For $y_t = x_{t+1}/r$ the expression reduces to $p_0 = (x_2 + rd_1 - x_1)/r^2$, which can be interpreted as leading up to a PEG-formula ($r = \sqrt{PEG^{-1}}$ where $PEG \equiv (p_0/x_1)/g_2$; see Ohlson

and Jeuttner-Nauroth (2005)). In both cases one loses a dimension: in the current case p_0 does not depend on x_1 and the increment $x_2 + r \cdot d_1 - x_1$ suffices for p_0.

[11] Yee (2005) develops broader insights about the weighted average function

$$p_0 = wy_0 + (1 - w)(y_1 + d_1)/R,$$

where y_t is either (expected) book value or capitalized earnings. Yee considers $p_0 = \sum_i w_i E_0[\tilde{q}_i]$, $\sum_i w_i = 1$ and q_i is equivalent to the value of a savings account at date zero. For example, q_i can equal $(R/r)x_0 - d_0$, $[x_3 + rd_2 + (R^2 - 1)d_1]/rR^2$, $(b_2 + d_2 + rd_1)/R^2$, $b_{-1}R - d_0$, and so forth. Note that one can go back in time as well as forward. Yee (2005) shows that such a representation is necessary and sufficient for DPI. Hence, given DPI, one obtains a description of all admissible reduced-form valuation functions. That said, the issue still remains what factors determine the weights and any construct of growth. To illustrate Yee's result, it can be combined with the specification $y_t = (b_{t+1} + d_{t+1})/R$ and Proposition 5. One obtains $q_1 = y_0$ and $q_2 = (b_2 + d_2 + rd_1)/R^2$, $w_1 = -\gamma/(R - \gamma)$.

[12] From Proposition VI it follows that if $r = x_1/p_0$ ($y_t = b_t$ and $\gamma = 1$), then $r = \sqrt{PEG^{-1}}$. The converse is false.

[13] Ozair (2003) provides most of this section's results.

[14] Because the ε_{2t}-distribution can be information-dependent, to add restrictions on the distribution of ε_{2t} such that, for all t, for sure, $v_{1t} > 0$, poses no problem.

[15] To prove this equivalence requires separate treatment of the two cases. Consider ID-1 first. Without loss of generality, assume full payout (i.e., $d_{t+\tau} = \tilde{x}_{t+\tau}$). With $d_{t+\tau} = \tilde{x}_{t+\tau}$ in place, $b_{t+\tau} = b_t$, for $\tau \geq 0$, which makes book value a constant. A direct inspection of ID-1 shows that there is

growth in expected earnings if and only if $\gamma > 1$. (It is assumed, of course, that $v_{1t} > 0$.) But from the last proposition it is readily seen that $\gamma > 1$ is also necessary and sufficient for $E_t[\Delta\widetilde{x}_{t+\tau} + r(\widetilde{x}_{t+\tau} - \widetilde{d}_{t+\tau})] > 0$, which proves the conclusion. Consider next ID-2. The reasoning is exactly the same, except that the condition $\gamma > 1$ is replaced by $\gamma > 0$. Thus it follows that the growth in expected earnings and the price-to-forward earnings ratio premium are implied by each other.