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OLIGOPOLY, DISCLOSURE AND EARNINGS MANAGEMENT*

by

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1. Introduction.

C. Michael Armstrong, former CEO of AT&T, has recently argued that accounting fraud at Worldcom was the cause of AT&T's *perceived* strategic failures, its inability to compete with Worldcom and, in the end, the decision to break up the company. More specifically, Armstrong has argued that Worldcom's fraudulently reported revenues, margins and costs drove AT&T's layoffs, cost cutting and a very unprofitable price war that left the company unable to service its debt. His view of the impact of Worldcom's accounting on competitors is also held by former Sprint CEO William Estry.[1] Motivated by these arguments, we develop a model designed to enhance our understanding of how firms can obtain a competitive advantage by biasing disclosures made in their financial statements.

We employ an incomplete information Cournot duopoly model in which each firm knows its own production costs but not its rival's. Each firm also provides a disclosure through, for example, its income statement. The firm's rival can use the disclosure to update its beliefs about the disclosing firm's production costs prior to competing in the product market. Our model differs from prior work on disclosure in incomplete information Cournot models because we assume that firms can provide biased reports. However, if they do so, they incur a cost of misreporting.[2] We show that when the firms know each other's cost of misreporting, there are no linear equilibria. However, when these costs are also private information, there are linear equilibria in which the firms bias their disclosures to gain a competitive advantage in the product market.

In particular, each firm's disclosure is designed to imply that its production costs are lower than they actually are. Further, because each firm uses all available information efficiently and fully understands their rival's incentives to bias its report, each adjusts its beliefs about its rival's production costs upward relative to the disclosure but, in equilibrium, still underestimates those costs. Consequently, each firm cuts its production below the full–information level causing the market price to rise and each firm's product market profits to increase, providing just the incentive needed for each firm to bias its disclosure. Said differently, in our model, each firm engages in

[1] See, for example, the recent book by Martin [2004], the former head of public relations at AT&T, or the articles by Searcy [2005], Blumenstein and Grant [2004] and McConnell et al [2002].

[2] The literature in this area includes Fried [1984], Shapiro [1986], Gal–or [1988, 1996], Darrough [1993], Raith [1996] and Vives [2002]. See, also, the survey by Verrecchia [2001]. When the authors in this literature focus on disclosure, the assumption is that the firms' disclosures are unbiased but that the firms can determine the amount of noise in the disclosure. Thus, if the firm wishes to disclose, it chooses not to add noise and if it wishes not to disclose, it chooses to add an infinite amount of noise.

earnings management that results in a downwardly biased estimate of its production cost.[3] Our analysis indicates that these effects are smaller when the firms compete in more profitable product markets or when they use reasonably similar technologies. Interestingly, while the effects are smaller in both cases, they differ in that firms engage in more (less) misreporting in the first (second) situation. Finally, when firms have different costs of misreporting, the firm with the lower costs introduces greater bias into its reports, produces more output than it would in the full information outcome and earns greater product market profits. Thus, our analysis supports Armstrong's view of the competitive impact of Worldcom's fraudulent accounting and suggests that it may be worth examining similar competitive environments for similar activity.

By focusing on equilibrium incentives to bias reports, our analysis suggests new empirical implications associated with the use of different earnings management techniques. First, as mentioned in footnote 3, only some techniques can effectively bias inferences about the firm's production costs. Thus, our model predicts that the use of these techniques is positively correlated with standard measures of profitability such as ROE. Further, we expect that firms that have similar production technologies will be less likely to employ these types of earnings management techniques. Thus, firms whose production is governed by physical or chemical processes or firms in mature industries are less likely to engage in this type of earnings management than firms in service industries or firms that have significantly larger portfolios of products. Finally, our analysis suggests that the increased use of deferred prosecution agreements (DPAs) is likely to lead to unexpected consequences. In particular, because such agreements dramatically increase the cost of misreporting, the competitors of firms operating under DPAs are likely to respond by increasing the amount of misreporting in order to gain/increase their competitive advantage in the product market.

The literature on the interaction between disclosures and product market competition was extended and synthesized in Darrough [1993] and subsequently generalized in Raith [1996]. The key difference between this literature and our model is that prior work required firms to make unbiased disclosures—firms could only alter the amount of noise in their disclosure—whereas we permit firms to bias their disclosures at a cost which leads to very different predictions. For Cournot competitors with private information about production costs, Darrough and others have

[3] Examples of this type of earnings management include fraudulent revenue recognition, aggressive cost capitalization, inclusion of operating costs in restructuring costs, inappropriately low estimates of the allowance for doubtful accounts or warranty expense, and certain types of "real" earnings management such as delaying expenditures. Examples of earnings management techniques that would not lead to biased inferences about production costs include channel stuffing, delayed write–downs of assets, and the "timely" sale of assets.

shown that firms prefer to precommit to disclosing their private information without adding any noise. Intuitively, adding noise results in the firm's rival choosing to produce more than it would if fully informed which lowers price and product market profits. If the disclosure decision is made after the firms learn their production costs then, because they are required to disclose truthfully, the standard unraveling story holds.[4] Our results complement the prior literature by showing that when firms are permitted to bias disclosures (even at a cost), they choose to do so and that this decision is affected by and affects competition in the product market.[5]

The remainder of the paper is organized as follows. Section 2 contains a description of our model of disclosure and product market competition and describes the equilibrium. In Section 3, we focus on symmetric equilibria and analyze asymmetric equilibria in Section 4. We conclude in Section 5.

2. The Model and Equilibrium.

We examine a two–stage game designed to capture the interaction between a firm's disclosure, earnings management and its competitive position in its product market. In the first stage, each of two firms chooses a disclosure without knowing its rival's choice and incurs a disclosure cost. Each firm makes this disclosure knowing both its own disclosure and production costs. As a result, the disclosure may provide information about the firm's cost of production and may therefore be of use to its rival when the two firms compete in their product market. In the second stage of the game, the firms observe both disclosures from the first stage and compete in the product market by choosing an amount of the homogeneous product to offer for sale without observing the output choice of its rival. We simplify the analysis by assuming that each firm has constant marginal costs of production and fixed costs are zero. Together, these imply that the firm's average costs of production equal its marginal costs.

More specifically, in the disclosure stage of our game, each firm chooses a disclosure which we interpret as a disclosure about the firm's average cost of production. Our objective is to capture

[4] See Christensen and Feltham [2002] for a nice summary of this literature. Interestingly, in a recent paper, Arya, Frimor and Mittendorf [2007] have shown that if the firms compete in multiple markets, the unraveling result may fail. In particular, they show that there can be a partial pooling equilibrium in which firm types that have high costs in one market and low costs in the other pool with firm types with low costs in the first and high costs in the second.

[5] The issue of whether the firms precommit to disclose does not arise substantively in our model because the firms have the ability to select a disclosure that is independent of their private information. Thus, even if the firms precommit to disclose, they have an available strategy that allows them to reveal none of their private information.

the idea that a firm's financial statement information (*e.g.*, an income statement) can be used to infer the firm's reported marginal/average costs. Further, since firms can make reporting decisions that either enhance or diminish reported earnings (both within the discretion afforded by GAAP and/or through fraud), we do not require that the firm disclose truthfully. Instead, we assume that if it does not disclose truthfully, it incurs a disclosure cost $h_i(s_i, c_i), \quad i = 1, 2$ where s_i is firm i's disclosure and c_i is its cost of production. To keep the analysis tractable, we assume that $h_i(s_i, c_i) = k_i(s_i - c_i) + (1/2)\varepsilon_i(s_i - c_i)^2$ with $\varepsilon_i > 0$. Notice that with this specification, firms that disclose truthfully incur no disclosure costs but firms that do not to disclose truthfully incur two types of costs. The first type (represented by the second term) imposes a quadratic cost of misreporting c_i—if the firm misreports ($s_i \neq c_i$), it incurs a cost that is proportional to the squared difference between its disclosure and its actual marginal cost of production.[6] As a result, it is symmetric in deviations from disclosing truthfully. The second type of disclosure cost (represented by the first term) is not symmetric and depends on the sign of k_i. Intuitively, if $k_i > 0$, then the firm's disclosure costs increase when its disclosure exceeds its actual marginal cost of production and may represent an added disadvantage from the market inferring that it is less competitive. Similarly, if $k_i < 0$, the firm's disclosure costs increase when its disclosure is smaller than its actual marginal costs and may represent an added disadvantage associated with a (perceived) weakened position in bargaining with employees or in dealing with regulators (Watts and Zimmerman [1986, 1990]).

In the second stage of the game, the two firms observe the first–stage disclosures (s_1, s_2) and compete in the product market. We model the product market using a fairly standard incomplete information, Cournot duopoly model where firms know their own cost of production but do not know their rival's costs.[7] More specifically, we assume that the market demand for the homogeneous product sold by the two firms is $P = a - Q$ where $Q = q_1 + q_2$ and units of output are normalized so that the slope coefficient of the demand function is 1. All information about market demand is common knowledge.

An important component of our model is the information structure. We assume that firm i

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6 Quadratic cost functions have become relatively common in the disclosure literature because they are particularly tractable. See, for example, Crawford and Sobel [1982], Fischer and Stocken [2001] and Chakraborty and Harbaugh [2007] who employ a quadratic cost function in their models of cheap talk as well as the models in Stocken and Verrecchia [2004], Fischer and Stocken [2004] and Guttman et al. [2006].

7 Prior work using a Cournot model with incomplete information about costs include Fried [1984], Shapiro [1986], Gal–or [1988, 1996], Darrough [1993] and Vives [2002]. See also the survey on disclosure by Verrecchia [2001].

knows its own disclosure costs (k_i) and its costs of production (c_i) but not its rival's disclosure costs (k_j) or its rival's costs of production (c_j). All other parameters are common knowledge. Thus, each firm has two pieces of private information, k_i and c_i, and does not know k_j and c_j.[8] We assume that each firm's priors are that its rival's disclosure costs are independent and normally distributed and that its rival's marginal costs of production are also normally distributed. A key feature of our model is that we assume that the covariance between the two firms' marginal costs is non–negative. Such an assumption implies that each firm learns about its rival's marginal costs when it learns its own marginal costs. More formally, we assume that k_1 and k_2 are independent, normally distributed random variables with zero means and variances $\sigma_{k_1k_1}, \sigma_{k_2k_2}$. Further, we assume that $c_i \sim N(\mathrm{E}[c_i], \sigma_{ii})$ for $i = 1, 2$ and that $\mathrm{Cov}[c_1, c_2] \equiv \sigma_{12} \geq 0$.

A key difference between our model and others in the literature is that we assume that each firm has two sources of private information, its disclosure costs and its cost of production. As a result, firm i's disclosure in the first stage is likely to depend on both components of its private information and therefore may provide its rival with information about c_i. If so, then the disclosure will affect how the firms compete in the product market.

As is standard in the literature, we focus on linear equilibria. In the first stage of the game, each firm knows its own disclosure and production costs. That is, firm i's first period information set is $y_i^1 = (c_i, k_i)$ and so we conjecture that each firm uses a strategy that is linear in the elements of its first stage information set. In the second stage of the game, each firm has observed the disclosures made in the first stage of the game and, as a result, firm i's information set is $y_i = (c_i, s_i, s_j)$. We include s_i but not k_i because, while firm i's own disclosure does not provide it with additional information, its rival uses s_i to infer c_i. Thus, firm i anticipates that q_j will depend on s_i and therefore conditions its own output choice, q_i, on s_i. Neither firm's output choice depends on k_i because neither uses its own disclosure to infer its own private information. As a result, we conjecture that, in a linear equilibrium,

$$\text{(C1)} \qquad \begin{aligned} s_1 &= D_0 + D_1 c_1 + D_2 k_1 \\ s_2 &= F_0 + F_1 c_2 + F_2 k_2 \\ q_1 &= N_0 + N_1 c_1 + N_2 s_1 + N_3 s_2 \\ q_2 &= M_0 + M_1 c_2 + M_2 s_1 + M_3 s_2. \end{aligned}$$

[8] Formally, we are assuming that each firm's type is two–dimensional, $t_i = (k_i, c_i) \in \mathcal{R}^2$.

To solve for a perfect Bayes equilibrium of the game, we begin by analyzing the second stage game which describes how the firms compete in the product market. Given our normality assumptions and our focus on linear equilibria, we make the additional conjecture that[9]

(C2)
$$\begin{aligned} \mathrm{E}[c_2 \mid y_1] &= \alpha_0 + \alpha_1 c_1 + \alpha_2 s_2 \\ \mathrm{E}[c_1 \mid y_2] &= \beta_0 + \beta_1 c_2 + \beta_2 s_1. \end{aligned}$$

Thus, in the second stage, firm i solves

(1)
$$\max_{q_i} \ \mathrm{E}[(a - Q)q_i - c_i q_i \mid y_i] \equiv \max_{q_i} \ aq_i - q_i^2 - \mathrm{E}[q_j \mid y_i]q_i - c_i q_i.$$

Note that the first stage disclosure costs have been ignored because, in the second stage, they are a sunk cost and thus do not affect the optimal output decision. The first order condition for this maximization problem is $0 = a - 2q_i - \mathrm{E}[q_j \mid y_i] - c_i$, which forms the basis for the following Proposition.[10]

Proposition 1: *If (C1) and (C2) hold, then there exists a linear equilibrium (q_1^*, q_2^*) to the production game with*

$$\begin{aligned} q_1^* &= N_0 + N_1 c_1 + N_2 s_1 + N_3 s_2 \\ q_2^* &= M_0 + M_1 c_2 + M_2 s_1 + M_3 s_2, \end{aligned}$$

where

$$\begin{aligned} N_0 &= (1/3)\left(a - \tfrac{(2-\beta_1)\alpha_0}{4-\alpha_1\beta_1} + \tfrac{2(2-\alpha_1)\beta_0}{4-\alpha_1\beta_1}\right) & M_0 &= (1/3)\left(a - \tfrac{(2-\alpha_1)\beta_0}{4-\alpha_1\beta_1} + \tfrac{2(2-\beta_1)\alpha_0}{4-\alpha_1\beta_1}\right) \\ N_1 &= -\tfrac{2-\beta_1}{4-\alpha_1\beta_1} & M_1 &= -\tfrac{2-\alpha_1}{4-\alpha_1\beta_1} \\ N_2 &= -(1/3)\alpha_2\left(\tfrac{2-\beta_1}{4-\alpha_1\beta_1}\right) & M_2 &= (2/3)\alpha_2\left(\tfrac{2-\beta_1}{4-\alpha_1\beta_1}\right) \\ N_3 &= (2/3)\beta_2\left(\tfrac{2-\alpha_1}{4-\alpha_1\beta_1}\right) & M_3 &= -(1/3)\beta_2\left(\tfrac{2-\alpha_1}{4-\alpha_1\beta_1}\right) \end{aligned}$$

There are two important features of the linear equilibrium described in Proposition 1 that we should highlight. First, each firm's equilibrium production strategy depends on its rival's disclosure, its actual production costs and on its own disclosure. Consistent with our informal explanation for why we included the firm's own disclosure costs in its information set, the (potentially) unintuitive feature is that the production strategy depends on its own disclosure despite the fact that the

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[9] In a linear equilibrium, we will have to compute all of the conjectured coefficients in (C1) and (C2).

[10] The second order condition is satisfied since the coefficient on q_i is negative.

firm knows its own cost of production and uses it rather than its own disclosure to estimate its rival's marginal cost of production. As we suggested above, the reason for this dependence is that firm i knows that its rival is using i's disclosure to make inferences about i's production costs (see equations (C2)) and thus i can infer that its rival's production strategy will depend on s_i. Since firm i's production strategy depends on its inference about firm j's production decision which firm i knows depends on s_i, i's production strategy "indirectly" depends on its disclosure because of the information its rival can extract from that disclosure. Second, the parameters describing the equilibrium production strategies (the M's and N's) depend on the coefficients in the conditional expectations described by (C2). Intuitvely, both firms are using all of their private information and their public disclosures from the disclosure stage of the game to infer as much as they possibly can about their rival's production costs.

Proposition 1 can also be used to derive each firm's equilibrium expected second stage profits. Substituting q_i^* into equation (1) shows that (as usual in Cournot games) firm i's equilibrium expected second stage profits are

$$\mathrm{E}[\pi_i^2 \mid y_i] = (a - c_i - q_i^* - \mathrm{E}[q_j^* \mid y_i])q_i = (1/2)(q_i^*)^2.$$

It will also be useful to have simplified expressions for $\mathrm{E}[q_j^* \mid y_i]$. This is most readily obtained from Proposition 1 by taking expectations:

$$\mathrm{E}[q_2^* \mid y_1] = M_0 + M_1\mathrm{E}[c_2 \mid y_1] + M_2 s_1 + M_3 s_2$$

$$\mathrm{E}[q_1^* \mid y_2] = N_0 + N_1\mathrm{E}[c_1 \mid y_2] + N_2 s_1 + N_3 s_2.$$

Substituting for $\mathrm{E}[c_i \mid y_j]$ and rearranging shows that each equation has the form implied by (C1):

$$\mathrm{E}[q_2^* \mid y_1] = A_0 + A_1 c_1 + A_2 s_1 + A_3 s_2 \tag{2}$$

$$\mathrm{E}[q_1^* \mid y_2] = B_0 + B_1 c_2 + B_2 s_1 + B_3 s_2. \tag{3}$$

Folding back to the first stage of the game, the disclosure stage, each firm chooses a disclosure, s_i, which we interpret as a disclosure about the firm's average (equivalently, in our model, marginal) cost of production. Recall that our interpretation is that the firm's income statement can be used to infer the firm's reported average costs. Given our distributional and information assumptions, in the first stage of the game, firm i chooses s_i to maximize its expected profits in the two–stage game overall:

$$\max_{s_i} \quad (1/2)\mathrm{E}[(q_i^*)^2 \mid y_i^1] - k_i(s_i - c_i) - \varepsilon_i(s_i - c_i)^2.$$

Substituting for the equilibrium outputs from Proposition 1, and maximizing with respect to s_1 (resp. s_2) yields,

$$s_1 = \left[\frac{1}{\varepsilon_1 - A_2^2}\right]\Big(-A_2\big(a - A_0\big) + \big(\varepsilon_1 + A_2(1 + A_1)\big)c_1 - k_1 - A_2A_3\mathrm{E}[s_2 \mid c_1, k_1]\Big) \tag{4}$$

$$s_2 = \left[\frac{1}{\varepsilon_2 - B_3^2}\right]\Big(-B_3\big(a - B_0\big) + \big(\varepsilon_2 + B_3(1 + B_1)\big)c_2 - k_2 - B_2B_3\mathrm{E}[s_1 \mid c_2, k_2]\Big). \tag{5}$$

Since k_1 and k_2 are not correlated with each other or with the firms' marginal costs of production, $\mathrm{E}[s_j \mid k_i, c_i] = \mathrm{E}[s_j \mid c_i]$. Equations (4) and (5) suggest that s_i is linear in c_i, k_i so long as the conditional expectations $\mathrm{E}[s_j \mid c_i]$ are linear in those variables too. Proposition 2 provides conditions under which the s_i^*'s are, in fact, linear resulting in a linear equilibrium to our two–stage game.

Proposition 2: *In every linear equilibrium, (q_1^*, q_2^*) are defined as in Proposition 1 and (s_1^*, s_2^*) satisfy*

$$s_1^* = D_0 + D_1c_1 + D_2k_1$$
$$s_2^* = F_0 + F_1c_2 + F_2k_2,$$

where

$$\mathrm{E}[c_2 \mid c_1] = \mu_0 + \mu_1c_1 \qquad \mathrm{E}[c_1 \mid c_2] = \delta_0 + \delta_1c_2$$

$$D_0 = \tfrac{1}{\varepsilon_1 - A_2^2}\Big[-A_2(a - A_0 + A_3F_0 + A_3F_1\mu_0)\Big] \qquad F_0 = \tfrac{1}{\varepsilon_2 - B_3^2}\Big[-B_3(a - B_0 + B_2D_0 - B_2D_1\delta_0)\Big]$$

$$D_1 = \tfrac{1}{\varepsilon_1 - A_2^2}\Big[A_2(1 + A_1) - A_2A_3F_1\mu_1 + \varepsilon_1\Big] \qquad F_1 = \tfrac{1}{\varepsilon_2 - B_3^2}\Big[B_3(1 + B_1) - B_2B_3D_1\delta_1 + \varepsilon_2\Big]$$

$$D_2 = -\Big(\tfrac{1}{\varepsilon_1 - A_2^2}\Big) \qquad F_2 = -\Big(\tfrac{1}{\varepsilon_2 - B_3^2}\Big)$$

We should also observe that in any linear equilibrium as described in Proposition 2,

$$\begin{aligned}\mathrm{E}[c_2 \mid c_1, s_1] = \mathrm{E}[c_2] &+ \left(\frac{(\sigma_{22}F_1^2 + \sigma_{k_2k_2}F_2^2)\sigma_{12} - \sigma_{12}\sigma_{22}F_1^2}{\sigma_{11}(\sigma_{22}F_1^2 + \sigma_{k_2k_2}F_2^2) - \sigma_{12}^2F_1^2}\right)(c_1 - \mathrm{E}[c_1]) \\ &+ \left(\frac{\sigma_{11}\sigma_{22}F_1 - \sigma_{12}^2F_1}{\sigma_{11}(\sigma_{22}F_1^2 + \sigma_{k_2k_2}F_2^2) - \sigma_{12}^2F_1^2}\right)(s_2 - F_0 - F_1\mathrm{E}[c_2])\end{aligned}$$

$$\begin{aligned}\mathrm{E}[c_1 \mid c_2, s_1] = \mathrm{E}[c_1] &+ \left(\frac{(\sigma_{11}D_1^2 + \sigma_{k_1k_1}D_2^2)\sigma_{12} - \sigma_{12}\sigma_{11}D_1^2}{(\sigma_{11}D_1^2 + \sigma_{k_1k_1}D_2^2)\sigma_{22} - \sigma_{12}^2D_1^2}\right)(c_2 - \mathrm{E}[c_2]) \\ &+ \left(\frac{\sigma_{11}\sigma_{22}D_1 - \sigma_{12}^2D_1}{(\sigma_{11}D_1^2 + \sigma_{k_1k_1}D_2^2)\sigma_{22} - \sigma_{12}^2D_1^2}\right)(s_1 - D_0 - D_1\mathrm{E}[c_1]),\end{aligned}$$

and a linear equilibrium requires that these coefficients match the α's and β's in equations (C2).

Unfortunately, there is no general result on the existence of a perfect Bayes equilibrium in this type of linear–normal model.[11] Further, finding a linear equilibrium in closed form for our model is very difficult because it requires solving a system of 24 non–linear equations for the unknown coefficients. While numerical solutions can be obtained (and we do so below), in the next subsection, we focus on symmetric equilibria and show that a linear equilibrium exists.

Before continuing, we should discuss the importance of our assumption that firm types are two–dimensional. The easiest way to see the role of this assumption is to consider the effect of assuming that both firms know the realizations of k_1 and k_2 (equivalently that $\sigma_{k_1 k_1} = \sigma_{k_2,k_2} = 0$). In this event, if the two firms adopt linear disclosure strategies, then each can invert their rival's strategy and infer their rival's production costs from their rival's disclosure. That is, if s_i is a linear function of the only information that firm j does not know, c_i, then firm j can infer the realized value of c_i from observing s_i. This turns the second–stage game into a game of complete information since both firms now know their rival's costs of production (and everything else is common knowledge) and we are able to prove the following result:

Proposition 3: *If k_1 and k_2 are common knowledge, there are no linear equilibrium in pure strategies.*

The key implication of assuming that k_1 and k_2 are common knowledge is that neither firm can employ a linear disclosure strategy without revealing its actual production costs and, as a result, there are no linear equilibria in pure strategies. To see why, note that each firm's marginal disclosure cost is approximately zero for small amounts of misreporting. Further, if firm i misleads j into believing that i's costs are lower than they actually are, it induces j to reduce its output which allows firm i to increase its own output (in anticipation of j's output choice). As a result, product market profits are shifted from firm j to firm i. Thus, in any potential equilibrium in which firm j believes that it can correctly infer firm i's cost of production, firm i has incentives to mislead thereby destroying the potential equilibrium. This analysis shows that our assumption that each firm's type is two–dimensional eliminates the possibility that each firm can infer its rival's cost of production from a linear disclosure strategy and, as a result, allows for the possibility of a linear

[11] With minor modifications to our distributional assumptions however, the existence of a trembling hand perfect Nash equilibrium ensures that there is a perfect Bayes equilibrium (see Fudenberg and Tirole [1993]). In particular, if each firm's private information is drawn from a finite set (so that the set of types in our game is finite), standard existence theorems ensure that there is a trembling hand perfect Nash equilibrium and therefore a perfect Bayes equilibrium in the modified game.

equilibrium.[12]

3. Symmetric Equilibria.

In this subsection, we focus on symmetric equilibria and in the following subsection, we explore the nature of asymmetric equilibria. In a symmetric equilibrium, both firms employ the same strategies—the same map from their private information to their actions. Thus, the equilibrium actions of the firms s_1, s_2 are determined by the same function of the firm's private information (c_1, k_1 and c_2, k_2 respectively) and the equilibrium production decisions q_1, q_2, are determined by the same function of each firm's information set (c_1, s_1, s_2 and c_2, s_1, s_2 respectively). That is, symmetry requires that $D_i = F_i$ and $\alpha_i = \beta_i$ for $i = 1, 2, 3$; $N_i = M_i$ for $i = 1, 2, 3, 4$ and $\mu_i = \delta_i$ for $i = 1, 2$. Further, we must assume that both firms are in the same "competitive position:" k_1 and k_2 must be drawn from the same distribution, c_1 and c_2 must be drawn from the same distribution and the firms must incur the same disclosure costs, $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon$. Finally, without loss of generality, we can simplify by assuming that all of the normally distributed random variables are standard normal with zero means and variances equal to 1.[13] Given this structure, we have

Proposition 4: *If ε is not too small, there is a symmetric linear perfect Bayes equilibrium in pure strategies.*

Intuitively, if the cost of misreporting is sufficiently small we cannot support a linear equilibrium because each firm's incentive to misreport is too great and thus cannot depend linearly on the firm's private information. However, if the cost of misreporting is not too small, then there is a linear perfect Bayes equilibrium in which each firm provides a disclosure. The disclosure allows the firm's rival to update its beliefs about the disclosing firm's production costs but does *not* allow it to infer exactly what those costs are. The update does, however, affect how the rival competes with the disclosing firm in the product market.

To better understand the properties of the disclosure and its effects on product market competition, we will generally do numerical comparative static analysis because of the complexity of the equations defining the symmetric equilibrium. Before that, however, we begin with an intuitive result that does not require numerical techniques.

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[12] In that sense, the role of the second dimension of a firm's type in our model is similar to the role of noise traders in Kyle models of market making (see Kyle [1998] or Bagnoli, Viswanathan and Holden [2001]).

[13] Equivalently, we can transform to standard normal random variables.

Proposition 5: *As the cost of misreporting gets large,*

(i) *the amount of misreporting declines,*

(ii) *each firm's estimate of its rival's cost of production becomes more accurate and*

(iii) *each firm's output approaches the output it would make in a full–information environment.*

Proposition 5 tells us that as the cost of misreporting gets large, the equilibrium in the product market converges to the complete information solution. The reason is that as the cost of misreporting gets large, the firms do less and less making their disclosures more informative. Thus, the uncertainty about the rival's cost dissipates and competition in the product market approaches the outcome when the firms are fully informed of their rival's cost of production. Interestingly, this result does not require that firms precommit to a disclosure or be required to disclose truthfully. Instead, as the cost of misreporting rises, firm behavior converges to the full information solution where each truthfully discloses its private information about its production costs.

To get a clearer picture of how the equilibrium varies with respect to the cost of misreporting, the correlation between the firm's costs of production and the size of the product market, we turn to numerical comparative static analyses. Since $h_i(s_i, c_i) = k_i(s_i - c_i) + (1/2)\varepsilon_i(s_i - c_i)^2$, a positive (negative) k_i produces marginal incentives for the firm to report $s_i < c_i$ $(s_i > c_i)$. Thus, we do all numerical analyses below assuming that the realized values of k_1 and k_2 are zero. Given this, we begin by analyzing the equilibrium described above in more detail and the effects of an increase in the cost of misreporting, ε.

Result 1: *In equilibrium, each firm's reported cost of production is smaller than its actual cost of production, each firm's output is smaller than the full–information quantity and each firm's estimate of its rival's costs is smaller than its rival's actual cost of production.*[14]

This result tells us that we should expect firms to bias their reported costs downward in an attempt to convince their rival that their costs are lower than they actually are. If they are successful, their rival will optimally reduce the amount it sells in the product market causing the price to be higher and resulting in greater profits for the misreporting firm. Each firm, in equilibrium, is successful because neither can perfectly extract its rival's cost of production from

[14] Further, combining Result 1 with Proposition 5 implies that when the cost of misreporting increases, all of these differences shrink with reported costs converging to actual costs, output choices converging to the full–information quantities and each firm's estimate of its rival's costs converging to its rival's actual costs.

its reported costs and while each understands the equilibrium and adjusts their expectation of their rival's cost taking into account the rival's incentive to report lower costs, the adjustment is only partial. Thus, in equilibrium, their expectation, conditional on everything they know, is still smaller than the rival's actual cost of production.

These results differ markedly from those in the prior literature (as described in Darrough [1993] or the generalization in Raith [1996]). Darrough proves that if the firm's disclosure is required to be a noisy but unbiased reflection of the firm's private information about its cost of production, in equilibrium, both firms would precommit to minimize the noise in the disclosure.[15] That is, they report their private information without bias. Other models permit firms to choose whether to disclose after learning their production costs but require that a firm choosing to disclose do so truthfully. This literature, summarized in Christensen and Feltham [2002], shows that the standard unraveling result applies—in equilibrium, every firm type discloses because if more than one type is pooled together, the type with the lowest production costs can increase profits by disclosing its costs and inducing its rivals to reduce the quantities that they offer for sale. In our model, the firms are not required to disclose truthfully but, if they report with a bias, they incur a cost associated with misreporting. Our analysis shows that when firms have the option to bias their disclosure (even at a cost), they will do so and that the effects are only partially accounted for by its rival.[16]

Interestingly, our analysis also shows how the opportunity to misreport earnings impacts the profits each firm earns in the product market. In particular, we have

Result 2: *In equilibrium, misreporting allows both firms to earn greater profits in the product market than they would if each had complete information about their rival's cost of production. Further, the profits the firms earn in the product market decline as the cost of misreporting increases.*

Intuitively, the type of earnings management we examine allows each firm to bias their disclosures in order to affect its rival's inference about the its cost of production. Result 1 indicates that the firms use this ability to misreport costs as being lower than they actually are. Upon

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[15] Formally, if the firm's private information is ψ_i, it is required to disclose $\hat{\psi}_i = \psi_i + \nu_i$ where ν_i is a normally distributed random variable with mean 0 and variance chosen by the disclosing firm. If the firm wishes to disclose its private information, it selects a variance of zero and if it wishes not to disclose, it selects a variance of infinity. In neither case, can the disclosing firm intentionally bias its disclosure.

[16] In this sense, our paper is in the spirit of Fisher and Verrecchia [2000] who extend disclosure models by assuming that the manager's objective function is not known to the market and show that the manager opts to bias his/her earnings report and that the market cannot fully adjust its expectation of firm value for the bias because of its uncertainty about the manager's objective function.

observing the disclosure, the rival updates its beliefs about the firm's costs using the disclosure and what it knows about the disclosing firm's incentives to bias but still ends up estimating that the firm's costs are lower than they actually are. As a result, the rival infers that the firm will be producing more than it actually does and responds by reducing the quantity that it chooses to produce. Consequently, earnings management results in both firms selling less than they would had they known their rival's costs but the associated price increase (from selling less) results in each firm earning greater profits than they would with complete information about its rival's costs. Further, Result 1 tells us that as the cost of misreporting rises, the magnitude of the bias in the firms' reports declines. Thus, since both firms' profits in the product market are greater the greater the equilibrium amount of earnings management, increases in the cost of misreporting lower the equilibrium amount of earnings management and therefore lower the profits each firm earns in the product market. Further, if the costs of misreporting are non–pecuniary or are incurred by management rather than the firm, Result 2 implies that earnings management actually increases the liquidation value of the firm.

The clearest empirical implications flow from associating changes in the cost of misreporting with changes in reporting regulations. For example, Section 302 of the Sarbanes–Oxley Act of 2002 (SOX) requires, among other things, that CEOs and CFOs (or persons performing equivalent functions) personally certify in each quarterly and annual report, including transition reports that, "...he or she has reviewed the report; based on his or her knowledge, the report does not contain any untrue statement of a material fact or omit to state a material fact necessary in order to make the statements made, in light of the circumstances under which such statements were made, not misleading with respect to the period covered by the report; based on his or her knowledge, the financial statements, and other financial information included in the report, fairly present in all material respects the financial condition, results of operations and cash flows of the issuer as of, and for, the periods presented in the report...."[17] Such certification increases the manager's costs of misreporting and would be represented by an increase in ε in our model. Thus, our model (Proposition 5 and Result 1) suggests the not surprising result that the impact of Section 302 is to reduce the amount of misreporting but also the more surprising results that: (i) each firm's estimates of their rival's costs increase and become more accurate and (ii) each firm produces

[17] See "Certification of Disclosure in Companies' Quarterly and Annual Reports," Securities and Exchange Commission 17 CFR PARTS 228, 229, 232, 240, 249, 270 and 274 [RELEASE NOS. 33-8124, 34-46427, IC-25722; File No. S7-21-02] RIN 3235-AI54, http://www.sec.gov/rules/final/33-8124.htm.

additional output. Thus, to the extent that misreporting of the type we consider is important to the economy, our analysis suggests that SOX, Section 302, should have expanded gross domestic product but reduced taxable profits and tax revenues.

More importantly, our analysis suggests that only certain types of misreporting are effective means by which a firm creates a competitive advantage in its product market. In particular, aggressive cost capitalization, including operating costs in restructuring costs, or selling previously written off inventory are all means of producing financial statements that lead the firm's rival to infer that the firm's costs are lower than they actually are. Further, changes in certain estimates can also produce the same result. For example, a firm that reduces the allowance for doubtful accounts (to increase earnings rather than because of changes in its customers' credit–worthiness) increases reported revenue relative to operating expenses and thus leads its rival to infer that the reporting firm's costs are lower than they actually are. Another example is reducing warranty expense estimates (without an associated change in the reliability of the product) that directly lowers reported operating expenses and misleads the firm's rival in a similar manner. Finally, certain methods of "real" earnings management such as delaying expenditures have the same effect. There are, however, a variety of "standard" earnings management techniques that would not create a competitive advantage of the type we examine. For example, aggressive revenue recognition or granting lenient credit terms (or, more aggressively, channel stuffing) all produce greater earnings (assuming gross margins are positive) but do not affect a rival's ability to use the firm's financial statements to infer its costs. Similarly, delays in writing down assets, over–reserving for contingencies or "timely" selling of assets are all means of managing earnings that also do not alter the rival's ability to infer costs. Thus, assuming that firms generally face similar incentives to engage in the second class of earnings management techniques, our analysis suggests that SOX will reduce the use of the first class of earnings management techniques relative to the second. As a result, our model predicts that the observed proportion of the first class of earnings management techniques among SEC enforcement actions will be smaller post–SOX.

Having analyzed the impact of earnings management on competition in the product market, we can expand our understanding of the incentives to manage earnings by examining how changes in the product market affect each firm's willingness to misreport costs.

Result 3: *When the firms compete in more profitable product markets, the magnitude of misreporting is greater but the bias in the rival's estimate of a firm's cost of production is smaller* *which, in turn, results in a smaller gain in equilibrium product market profits relative to the full information level of profit.*

Intuitively, Result 3 shows how the competitive environment in a firm's product market affects its incentives to engage in the type of earnings management we analyze. In particular, firms that compete in more profitable product markets (markets with larger demand intercepts) bias their reported earnings more because the benefits from inducing a rival to reduce output are greater. What may be less obvious is that the induced bias in the rival's expectation of the firm's cost is smaller. The reason for this is that we are holding the cost of misreporting, ε, constant. As the profitability of the product market increases, each firm reports ever smaller costs which increases the cost of misreporting. Thus, as product market profitability increases, the increase in the amount of misreporting is tempered by the increasing costs of misreporting which, in net, allows its rival to better estimate the firm's true cost of production. Since the estimate is better, the impact of misreporting on the rival's output choice declines and the equilibrium outcome converges to the full information solution.

An immediate implication of Result 3 is that both reported (actual) earnings are positively (negatively) correlated with the magnitude of misreporting (of the type we examine). As a result, our analysis predicts that most measures of profitability (*e.g.*, Return on Equity, Return on Invested Capital, Return on Sales, etc.) should be positively correlated with the magnitude of misreporting but negatively correlated after adjusting for the amount of misreporting. Alternatively, Result 3 implies that firms in highly profitable markets should be those firms that are more often engaged in and/or engage in more misreporting than firms in less profitable markets.

Result 4: *The more information a firm can extract about its rival's cost from knowing its own cost, the smaller is the magnitude of the misreporting, the smaller is the induced bias in the rival's estimate of the firm's cost and the smaller is the deviation from the full information level of output.*

The amount of information a firm can extract about its rival's costs from knowing its own costs is "measured" by the covariance between the costs, σ_{12}. In particular, the greater the covariance, the greater is the reduction in the firm's remaining uncertainty about its rival's cost after observing its own costs. The reduction in this uncertainty reduces the effectiveness of misreporting by offering a biased report and the result is that the product market equilibrium is closer to the full information outcome than it would be had the firms' costs been less highly correlated.

Result 4 suggests that firms with more similar technologies, such as those controlled by physical or chemical processes, should be expected to be less likely to engage in misreporting and, if they do so, to do so in smaller amounts than firms whose technologies are more likely to be very different (*e.g.*, service industries). We would also expect that firms in mature industries such as the auto or steel industries would also be less likely to engage in (or to do relatively less) misreporting than firms in industries with large portfolios of products. The latter are more likely to be in industries in which rivals have a more difficult time inferring its production costs from information about their own due to the large portfolio of products. One may potentially be able to test this empirically by relating misreporting to the number of business segments in the firm.

In summary, Results 1, 2, 3 and 4 show the full effect of firms exercising the option to bias their reported costs. Results 1 and 2 show how a firm's decision to bias its disclosure impacts the competition in its product market and Results 3 and 4 show the feedback effect—how the competitive environment in the firm's product market affects its incentives to manage earnings.

4. Asymmetric Equilibria.

The results in the previous subsection provide significant new insight into the relationship between product market competition and misreporting designed to bias rival's inferences about the reporting firm's cost of production. The one feature that cannot be explored when the focus is on symmetric equilibria is the impact of differential costs of misreporting.

To do so, recall that in the prior analysis, we took ε_i to be the cost of misreporting and implicitly treated it as either non–random or as the expected cost of misreporting.[18] The advantage of interpreting this as an expected cost is that there are now good reasons why firms would face different expected costs of misreporting. Not only might firms differ in the likelihood that they are examined for misreporting; it is also possible that firms are run by managers who differ in their concern about the impact of being detected misreporting. This situation appears to apply in the AT&T, Worldcom example cited in the introduction. To explore the effect of firms facing different costs of misreporting, we turn to an examination of asymmetric equilibria. We again apply numerical techniques because of the difficulties in solving the large system of non–linear equations that describe the equilibrium in our model. The key difference is that we relax the assumption

[18] If we assume that the cost of misreporting is uncorrelated with the firm's private information, then because the firm's payoffs are linear in that cost, we could take expectations with respect to it and simply replace the random variable with its expectation in the expressions derived above.

that $\varepsilon_1 = \varepsilon_2$ by assuming, without loss of generality, that $\varepsilon_2 > \varepsilon_1$. And, as in the prior subsection, we do all numerical analyses assuming that the realized values of k_1 and k_2 are zero.

Result 5: *As ε_2 rises relative to ε_1:*

(i) *the amount of misreporting by the low cost misreporter increases while the amount of misreporting by the high cost misreporter declines,*

(ii) *the bias in the estimate of the low cost misreporter is greater but both converge to zero as ε_2 rises*

(iii) *the low cost misreporter produces more output than the full information output while the high cost misreporter produces less and*

(iv) *the low cost misreporter earns greater product market profits than the high cost misreporter does and the difference increases as ε_2 increases.*

Result 5 describes how differences in the costs of misreporting affect competition. In particular, the greater the difference in the firms' costs of misreporting, the larger (smaller) is the amount done by the low cost (high cost) misreporter. Thus, holding the lower cost of misreporting constant, Result 5 suggests that increases in the higher cost firm's costs motivates its rival to do more misreporting. In fact, Result 5 says even more: If the low cost rises slower than the high cost, the direct effect of the cost increase on the low cost misreporter (to reduce the amount of misreporting) is overwhelmed by the indirect, competitive effect of the increase in its rival's cost of misreporting. Further, the lower cost of misreporting offers the firm a competitive advantage in the product market which it exploits by selling more output than its rival and earning greater profits.

Result 5 also suggests some empirically testable implications. First, it may be reasonable to assume that firms that were "forced" to replace upper management in the wake of a misreporting scandal are likely to face a competitive response from the firm's rivals which results in the rivals engaging in more misreporting, increasing production and shifting profits away from the firm that changed upper level management. Second, it suggests a potential risk from using deferred prosecution agreements in misreporting cases.[19] Such agreements increase (potentially dramatically) the firm's cost of misreporting and our Result 5 suggests that such agreements will produce unwanted competitive responses from the firm's rivals. In particular, the agreements will motivate the firm's

[19] Recent examples of the use of such agreements regarding misreporting of the type we examine include the Bristol–Myers Squibb case (www.usdoj.gov/usao/nj/press/files/pdffiles/deferredpros.pdf); the PNC Financial case (http://www.usdoj.gov/opa/pr/2003/June/03_crm_329.htm); the Computer Associates case (http://www.sec.gov/litigation/litreleases/lr18891.htm); and the AOL case (http://www.usdoj.gov/opa/pr/2004/December/04_crm_790.htm).

rivals to increase the amount of misreporting they do.

5. Conclusion.

In the past few years C. Michael Armstrong, former CEO of AT&T, has argued that AT&T's *perceived* strategic failures, its inability to compete with Worldcom and the decision to break the company up were the result of accounting fraud at Worldcom. He suggests that Worldcom's "...revenues were false, margins were false, their costs were false..." and that this resulted in layoffs, cost cutting and finally, the decision to break up AT&Tin order to service its debt. Former Sprint CEO William Esrey suggests that his company also struggled with its inability to match Worldcom's performance and noted that: "It never dawned on us the base of their pricing was fraud."[20] Motivated by these arguments, in this paper, we develop a model that illustrates how firms can obtain a competitive advantage by biasing disclosures in financial statements.

We examine an incomplete information Cournot duopoly model in which the firms know their own production costs but not their rival's. In our model, firms provide a disclosure (*e.g.*, an income statement) that its rival can use to update its beliefs about the disclosing firm's production costs prior to competing in the product market. Our model differs from the prior literature in that we allow firms to provide biased reports but, if they do so, they incur a cost of misreporting.[21] We show that when these costs of misreporting are common knowledge, there are no linear equilibria to our disclosure/competition model because the linear disclosure strategy can be inverted to correctly infer the rival's production costs.

However, when each firm is uncertain about their rival's cost of misreporting and these costs are not too small, there is a linear equilibrium in which the firms bias their disclosures in order to gain a competitive advantage in the product market.[22] In particular, they bias their reported costs downward. Further, even though the firms use all available information efficiently, fully understand each other's incentives and adjust their beliefs about their rival's costs upward, they

[20] See, for example, the recent book by Martin [2004], the former head of public relations at AT&T, or the articles by Searcy [2005], Blumenstein and Grant [2004] or McConnell et al [2002]. Esrey is quoted in Searcy [2005].

[21] The prior literature on disclosure and Cournot competition with imperfect cost information assumes that the firms are required to provide unbiased reports of their costs but can choose to add varying amounts of noise. The main result in this literature is that the firms choose to disclose without noise (Darrough [1993] or Raith [1996] among others).

[22] This result is reminiscent of Fisher and Verrecchia [2000]. They show that the introduction of uncertainty about the manager's objective function alters the standard results in the voluntary disclosure literature in that the manager in their model biases his/her earnings report and that the market cannot fully adjust its expectation of firm value for the bias because of its uncertainty about the manager's objective function.

still underestimate those costs. As a result, each cuts production relative to the full information level and each earns greater product market profits. Interestingly, these effects are smaller in more profitable product markets even though the magnitude of misreporting increases. They are also smaller when the firms use more similar production technologies. Finally, we show that when firms have different (expected) costs of misreporting, the bias in the low cost misreporter's disclosure is greater than its rival's and it produces more output than the full information quantity.

By focusing on competitive motives to bias reports, our analysis suggests some new empirical implications. To see why, note that aggressive cost capitalization, fraudulent revenue recognition, inappropriate estimates of the allowance for doubtful accounts or warranty expense are all examples of (one class of) earnings management techniques that lead to underestimates of the firm's production costs whereas other standard earnings management techniques (the second class) such as channel stuffing, delayed write–downs of assets or "timely" sales of assets do not lead to mis–estimated production costs. Since incentives to use the second class of earnings management techniques are likely to be independent of the incentives to employ the first class, our model predicts that regulatory increases in the cost of misreporting (*e.g.*, Section 302 of the Sarbanes–Oxley Act of 2002) will result in a change in the distribution of observed earnings management techniques (as, for example, among SEC enforcement actions). In particular, we expect a reduction in the use of the first class of earnings management techniques relative to the use of the second class. Second, our analysis suggests that standard measures of profitability will be positively correlated with the use of the first class of earnings management techniques. Third, firms with more similar technologies (*e.g.*, those for whom production is governed by physical or chemical processes or those used in mature industries) will be less likely to employ this type of earnings management than firms with less similar technologies (*e.g.*, service industries or firms that produce a large variety of different products).

Finally, when we examine asymmetric equilibria in which one firm's costs of misreporting are greater, we find that the results support Armstrong's assertions about the impact of Worldcom's accounting fraud on its competitors. Interestingly, the analysis also suggests that the more frequent use of deferred prosecution agreements may lead to unexpected consequences—providing the firm's competitors with incentives to misreport so as to bias estimates of their production costs down.

6. Appendix.

Proposition 1: *If (A1) holds, then there exists a linear equilibrium* (q_1^*, q_2^*) *to the production game with*

$$q_1^* = N_0 + N_1 c_1 + N_2 s_1 + N_3 s_2$$
$$q_2^* = M_0 + M_1 c_2 + M_2 s_1 + M_3 s_2,$$

where

$$N_0 = (1/3)\left(a - \tfrac{(2-\beta_1)\alpha_0}{4-\alpha_1\beta_1} + \tfrac{2(2-\alpha_1)\beta_0}{4-\alpha_1\beta_1}\right) \qquad M_0 = (1/3)\left(a - \tfrac{(2-\alpha_1)\beta_0}{4-\alpha_1\beta_1} + \tfrac{2(2-\beta_1)\alpha_0}{4-\alpha_1\beta_1}\right)$$

$$N_1 = -\tfrac{2-\beta_1}{4-\alpha_1\beta_1} \qquad M_1 = -\tfrac{2-\alpha_1}{4-\alpha_1\beta_1}$$

$$N_2 = -(1/3)\alpha_2\left(\tfrac{2-\beta_1}{4-\alpha_1\beta_1}\right) \qquad M_2 = (2/3)\alpha_2\left(\tfrac{2-\beta_1}{4-\alpha_1\beta_1}\right)$$

$$N_3 = (2/3)\beta_2\left(\tfrac{2-\alpha_1}{4-\alpha_1\beta_1}\right) \qquad M_3 = -(1/3)\beta_2\left(\tfrac{2-\alpha_1}{4-\alpha_1\beta_1}\right)$$

Proof: To find a linear equilibrium, compute using the conjectured equilibrium strategies,

$$\mathrm{E}[q_2 \mid y_1] = M_0 + M_1 \mathrm{E}[c_2 \mid y_1] + M_2 s_1 + M_3 s_2$$
$$\mathrm{E}[q_1 \mid y_2] = N_0 + N_1 \mathrm{E}[c_1 \mid y_2] + N_2 s_1 + N_3 s_2.$$

Substituting equations (A1),

$$\begin{aligned}
\mathrm{E}[q_2 \mid y_1] &= M_0 + M_1(\beta_0 + \beta_1 c_1 + \beta_2 s_2) + M_2 s_1 + M_3 s_2 \\
&= (M_0 + M_1\beta_1) + M_1\beta_1 c_1 + M_2 s_1 + (M_1\beta_2 + \beta_3) s_2 \\
&= A_0 + A_1 c_1 + A_2 s_1 + A_3 s_2; \qquad \text{(A1)} \\
\mathrm{E}[q_1 \mid y_2] &= N_0 + N_1(\alpha_0 + \alpha_1 c_2 + \alpha_2 s_1) + N_2 s_1 + N_3 s_2 \\
&= (N_0 + N_1\alpha_0) + N_1\alpha_1 c_2 + (N_1\alpha_2 + N_2) s_1 + N_3 s_2 \\
&= B_0 + B_1 c_2 + B_2 s_1 + B_3 s_2. \qquad \text{(A2)}
\end{aligned}$$

Substituting into the expressions for q_i for $i = 1, 2$ derived in the text,

$$\begin{aligned}
q_1 &= (1/2)(a - c_1) - (1/2)(A_0 + A_1 c_1 + A_2 s_1 + A_3 s_2) \\
&= (1/2)(a - A_0 - (1 + A_1) c_1 - A_2 s_1 - A_3 s_2); \qquad \text{(A3)} \\
q_2 &= (1/2)(a - c_2) - (1/2)(B_0 + B_1 c_2 + B_2 s_1 + B_3 s_2) \\
&= (1/2)(a - B_0 - (1 + B_1) c_2 - B_2 s_1 - B_3 s_2). \qquad \text{(A4)}
\end{aligned}$$

In equilibrium, the expectation of (A3) (resp. (A4)) must coincide with our conjectures. So, taking expectations of (A3) with respect to firm 2's information set (of (A4) with respect to firm 1's information set),

$$\mathrm{E}[q_1 \mid y_2] = (1/2)(a - A_0 - (1 + A_1)\mu_0 - (1 + A_1)\mu_1 c_2 - [(1 + A_1)\mu_2 - A_2] s_1 - A_3 s_2)$$
$$\mathrm{E}[q_2 \mid y_1] = (1/2)(a - B_0 - (1 + B_1)\xi_0 - (1 + B_1)\xi_1 c_1 - B_2 s_1 - [(1 + B_1)\xi_2 + B_3] s_2.$$

The proof is completed by matching the coefficients in equations (A1) and (A3) and equations (A2) and (A4), solving the resulting system of equations for the A's and B's and then solving for the M's and N's. ∎

Proposition 2: *In every linear equilibrium, (q_1^*, q_2^*) are defined as in Proposition 1 and (s_1^*, s_2^*) satisfy*

$$s_1^* = D_0 + D_1 c_1 + D_2 k_1$$
$$s_2^* = F_0 + F_1 c_2 + F_2 k_2,$$

where

$$\mathrm{E}[c_2 \mid c_1] = \mu_0 + \mu_1 c_1 \qquad \mathrm{E}[c_1 \mid c_2] = \delta_0 + \delta_1 c_2$$

$$D_0 = \tfrac{1}{\varepsilon_1 - A_2^2}\Big[-A_2(a - A_0 + A_3 F_0 + A_3 F_1 \mu_0)\Big] \qquad F_0 = \tfrac{1}{\varepsilon_2 - B_3^2}\Big[-B_3(a - B_0 + B_2 D_0 - B_2 D_1 \delta_0)\Big]$$

$$D_1 = \tfrac{1}{\varepsilon_1 - A_2^2}\Big[A_2(1 + A_1) - A_2 A_3 F_1 \mu_1 + \varepsilon_1\Big] \qquad F_1 = \tfrac{1}{\varepsilon_2 - B_3^2}\Big[B_3(1 + B_1) - B_2 B_3 D_1 \delta_1 + \varepsilon_2\Big]$$

$$D_2 = -\Big(\tfrac{1}{\varepsilon_1 - A_2^2}\Big) \qquad F_2 = -\Big(\tfrac{1}{\varepsilon_2 - B_3^2}\Big)$$

Proof: To find a linear equilibrium, compute using the conjectured equilibrium strategies,

$$\mathrm{E}[s_2 \mid c_1] = F_0 + F_1 \mathrm{E}[c_2 \mid c_1]$$
$$\mathrm{E}[s_1 \mid c_2] = D_0 + D_1 \mathrm{E}[c_1 \mid c_2],$$

where $\mathrm{E}[k_j \mid k_i, c_i] = \mathrm{E}[k_j \mid c_i] = \mathrm{E}[k_j] = 0$ by independence and our assumption that $\mathrm{E}[k_i] = 0$ for $i = 1, 2$. Further, since c_1, c_2 are joint normally distributed, $\mathrm{E}[c_2 \mid c_1] = \mu_0 + \mu_1 c_1$ and $\mathrm{E}[c_1 \mid c_2] = \delta_0 + \delta_1 c_2$. Substituting into equations (4) and (5) and using the definitions of the A's and B's ,

$$s_1 = \left[\frac{1}{\varepsilon_1 - A_2^2}\right]\Big[-A_2(a - A_0 + A_3 F_0 + A_3 F_1 \mu_1) + \big(A_2(1 + A_1) - A_2 A_3 F_1 \mu_1 + \varepsilon_1\big)c_1 - k_1\Big]$$
$$s_2 = \left[\frac{1}{\varepsilon_2 - B_3^2}\right]\Big[-B_3(a - B_0 + B_2 D_0 + B_2 D_1 \delta_0) + \big(B_3(1 + B_1) - B_2 B_3 D_1 \delta_1 + \varepsilon_2\big)c_2 - k_2\Big].$$

Since both are of the form assumed, the linear equilibrium is obtained by setting the coefficients in these equations equal to the F's and D's respectively. ∎

Proposition 3: *If k_1 and k_2 are common knowledge, there is no linear equilibrium in pure strategies.*

Proof: Suppose not. Then there exist strategies described by (C1) that form a linear perfect Bayes equilibrium. In such an equilibrium, firm i can invert the strategy defining its rival's choice of s_j and infer c_j. Thus, the second–stage game is now a game of complete information whose solution is well–known: $q_i^* = (1/3)(a - 2c_i + c_j)$ and equilibrium profits are $(q_i^*)^2$, for $i = 1, 2$. Next, consider a deviation in which firm i reports $\hat{s}_i$ which leads firm j to infer that firm i's cost of production is $\hat{c}_i \neq c_i$. Thus, $q_i = (1/3)(a - c_i - \hat{c}_i - c_j)$ and $q_j = (1/3)(a - 2c_j - \hat{c}_i)$. Since firm i's profits are q_i^2, they are decreasing in $\hat{c}_i$. As a result, if firm i deviates to $\hat{c}_i < c_i$, its profits in the product market increase which, for small deviations, exceed the costs of misreporting making firm i's deviation profitable and showing that there is no linear perfect Bayes equilibrium in pure strategies when k_1 and k_2 are common knowledge. ∎

Proposition 4: *If ε is not too small, there is a symmetric linear perfect Bayes equilibrium in pure strategies.*

Proof: In a symmetric equilibrium, $D_i = F_i$ and $\alpha_1 = \beta_i$ for $i = 1, 2, 3$; and $N_i = M_i$ for $i = 1, 2, 3, 4$ and $\mu_i = \delta_i$ for $i = 1, 2$. After matching coefficients, we have 12 non–linear equations in 12 unknowns describing the coefficients of the firms' strategies. By iterative substitution, one obtains a a system of two equations in two unknowns, D_1, D_2 (resp. F_1, F_2). One equation is quadratic in D_1 and has real roots when $\varepsilon_1 (= \varepsilon_2)$ is not too small. Solving for the roots and substituting into the other equation produces an 18^{th} order polynomial in D_2 with 87 sign changes. Thus, by Descartes' Rule of Signs (see, for example, Levin [2002] and the references therein), there is at least one real root and thus at least one symmetric linear equilibrium in pure strategies. ∎

Proposition 5: *As the cost of misreporting gets large, the amount of misreporting declines, each firm's estimate of its rival's cost of production becomes more accurate and each firm's output approaches the output it would make in a full–information environment.*

Proof: Suppose not, then in equilibrium, $\sigma_i - c_i > \xi_i > 0$ for $i = 1, 2$ for all ε. Since $\mathrm{E}[\pi_i \mid k_i, c_i] = \mathrm{E}[(q_i^*)^2 \mid k_i, c_i] - h_i(s_i, c_i)$ and $\mathrm{E}[(q_i^*)^2]$ is bounded from above by the firm's monopoly profits, $[(a - c_i)/2]^2$, if $\sigma_i - c_i > \xi_i > 0$, then there is a ε large enough so that $[(a - c_i)/2]^2 - h_i(s_i, c_i) < 0$. Thus, if $\sigma_i - c_i > \xi_i > 0$, there exists a large enough ε so that in the conjectured equilibrium, firm i's expected profits are negative. Since firm i could deviate to $s_i = c_i$ and earn non-negative profits, such a deviation is profitable and we have shown that in every linear perfect Bayes equilibrium, $\sigma_i - c_i > \xi_i > 0$ for all ε must be false. Given this, the difference $s_i - c_i$ must become arbitrarily small as ε gets large which implies that $\mathrm{E}[c_i \mid c_j, s_i, s_j] - c_i$ also becomes arbitrarily small as ε becomes arbitrarily large. Finally, since q_i^* is a linear function of c_i and $\mathrm{E}[c_j \mid c_i, s_i, s_j]$ in equilibrium, it too converges to the complete information Cournot output as ε becomes arbitrarily large. ∎

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