

# The Economics of the Bank and of the Loan Book

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Published by:

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## Introduction

Over the last decade there have been two major developments in commercial banking: the rapid growth of primary and secondary markets for trading credit risk, and active portfolio management of the bank's loan book. These developments coincide with a long-term change in the perception of the economics of commercial banking. Banks were once viewed as originating relatively safe assets, and earning money by the difference between their short term funding rate and their lending rates. Earnings came from assets. The bank and the portfolio were largely indistinguishable from each other.

Today, banks are viewed as originating riskier assets. Their funding rates are competitive market rates. The differences between funding and lending rates are mostly viewed as compensation for risk. Earnings are primarily generated by activities that explicitly or implicitly earn fees. The portfolio is viewed as a tool to support the bank's activities.

In this new view, banks earn money from loans by their underwriting or distribution activities. These earnings are represented by the difference in value between the funds lent and the claim created on the borrower. Those earnings can be achieved immediately, via selling the loan, or subsequently, by holding the loan until it matures. However, in the latter case, it is difficult to separate from the subsequent cash flows which ones represent the earnings to underwriting and which the earnings to the portfolio itself.

This problem has been partially addressed by RAROC models. The underlying intent of such models was to determine the profitability of a loan at the time of origination. However, the definition of profitability was based upon meeting internal hurdle rate returns for capital, without regard for whether the capital was deployed against the portfolio or against the non-portfolio activities of the bank.

It is now better understood that the profitability of a loan can be measured more accurately and more straightforwardly by decomposing the performance of the loan into two parts, one attributable to the underwriting activity, and one attributable to the subsequent performance of the loan. The underwriter's revenue is determined as the difference between the funds extended and the value of the loan held by the bank. The value of the loan is based primarily upon external market valuations of similar instruments, adjusted to reflect the particular characteristics of the loan. The underwriter's profitability is this revenue, minus the costs of the underwriting operation.

The second aspect of loan profitability is due to portfolio management. Subsequent to origination, the loan will change in value as external market values change, and as the credit quality of the borrower changes. These changes produce the performance of the portfolio. The appropriate standard for evaluating the profitability of the portfolio is relative to the performance of a well-constructed portfolio formed from the same universe of potential assets.

This "micro" decomposition of profitability for a single loan can be extended to a "macro" decomposition of the bank as a whole, by separating the portfolio and portfolio

management activities of the bank from the underwriting and non-portfolio services of the bank. This decomposition is very useful in understanding bank performance, as these two parts of the bank have very different characteristics and capital structures.

The remainder of this paper explores the issues raised above. The first part goes into greater detail on the “macro” decomposition of the bank into “portfolio” and “franchise”. The second part looks at the decomposition of loan revenue into “underwriting” and “portfolio” components, and their relationship to RAROC measures, with a discussion on loan valuation approaches. The third part explores the meaning of “mark to market” in the context of the credit portfolio. A brief summary concludes the paper.

A lengthy appendix addresses technical issues around actual loan valuation. The primary motive is to exposit the existing state of the art and, thus, to establish the feasibility of the approaches described in the paper.

## Franchise and Portfolio

Consider the balance sheet of a large bank, from the standpoint of accounting. Most of the assets on the balance sheet are financial claims, with a relatively small amount of depreciated real assets, as well as intangibles such as goodwill. In this perspective, the portfolio is the dominant aspect of the bank, and the implication is that bank performance flows primarily from portfolio performance.

If we contrast this accounting view with a market value based perspective, the resulting picture of the bank looks different in some significant ways. To get the market view, we need an alternative way to look at the bank's assets. We can achieve this by shifting our focus from the asset side of the balance sheet to the liability side. The market value of the bank's liabilities must equal the market value of the bank's assets; if we owned all the liabilities, we would have an unencumbered claim on all of the bank's assets.

We can get a decent approximation to this market view if we take the bank's liabilities to have market values close to their book values, except for the equity, where we can substitute the market value for the book value. When we do this, we discover that many banks have assets whose book value is considerably less than their market value.

Which assets on the balance sheet are the ones that are worth more than their book values? If we look at the loan book, it would be surprising if it were worth much in excess of its book value. Most individual loan values do not exceed par, and those that do, do so only by a small amount.

The missing market value is attributable to the bank's non-portfolio business activities. We call this the “franchise”. The franchise is a large, somewhat diversified service business. It represents the underwriting, distribution, fee services, and retail distribution activities of the bank. It is the business we would see if the bank did not retain any of the loans it originated.

The following chart illustrates this decomposition for a generic large bank in the United States. The market values are shown to the left of the each of the accounting balance sheets.

## Large Bank Example

Portfolio			Franchise		
Loans 364	ST 500	} <b>Market Value = \$577B</b>	Premises 7	ST 25	} <b>Market Value = \$94B</b>
Secs 83	LT 37		Intgbles 18	LT 26	
Other 130	Equity 40		Other 31	Equity 5	
Total 577			Total 56		

The balance sheet has been decomposed simply by putting all the financial claims into the portfolio, and allocating an appropriate mixture of debt and equity funding against those portfolio assets. The remainder of the assets is assigned to the franchise, along with the remainder of the liabilities and equity.

The market value decomposition is obtained simply by assuming that the portfolio's market value equals its book value, and assigning the remainder of the book value to the franchise. Although an approximation, this gives a good idea of the appropriate magnitudes.

Note that the franchise is worth close to \$100 billion. Most large bank franchises have market values in the range of \$30 billion to \$100 billion. These are very large service businesses.

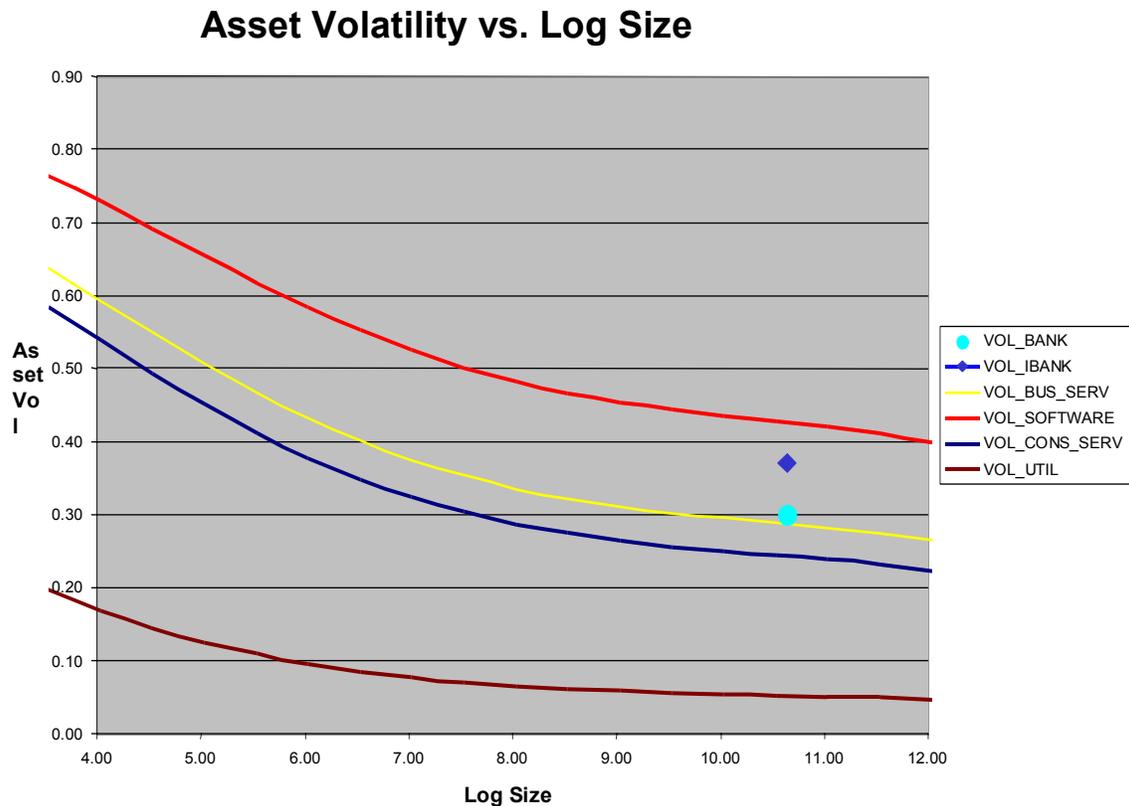
The same decomposition could be applied to a large investment bank, or to a large universal bank, with similar conclusions.

What else do we know about these two parts of the bank? In general, the portfolio has very little risk. For most banks, on a funded basis, the portfolio volatility is about 2% or less per annum. In other words, a one standard deviation move in the value of the portfolio, over a year, would be about 2%. The distribution of outcomes is not symmetric; the possibility of a large down move is economically significant, whereas large up moves are essentially impossible.

The risk of the portfolio can be determined very precisely by a detailed analysis of the portfolio, taking into account the risk of each individual asset, as well as the interrelationships of those risks.

The low volatility means that considerable leverage can be used to finance these assets, and leverage in excess of 90% is common.

The franchise has quite different characteristics. As a service business, it has approximately symmetric upside and downside risks. The risks cannot be readily ascertained from a micro analysis of the component businesses, but can only be approximately measured in the aggregate. Our best estimate of the volatility of the franchise is around 30% per annum for commercial banks and somewhat higher for investment banks. To get some idea of the significance of these values, we can contrast them with the typical volatilities that we observe for businesses of the same size in other industries. In the chart below, the smooth line represents the typical volatility as function of size for other industries; the points represent the values for commercial and investment banks.



For commercial banks, their volatility is consistent with equally large stand-alone non-financial business service companies. For investment banks, this volatility lies between those of business service companies and technology companies like software firms. In both cases, these volatilities are above average considering all firms of similar size.

If we look at the capital structure on a market value basis, we discover that unlike the portfolio, the franchise is largely unleveraged, with 20% leverage or less being typical. This is consistent with the relatively high risk profile of the franchise.

The upside potential of the bank resides almost entirely in the franchise. A bank creates franchise value when it figures out how to intermediate more efficiently and build scale as a result. For example, streamlining origination or distribution, and gaining market share results in increased franchise value.

However, there is an important caveat in this picture. Analysts and investors are constantly looking for banks that can produce a steady stream of earnings from their franchise. A bank that is simply taking more risk can produce the false appearance of a steady stream of earnings. Eventually, such risks end up producing significant portfolio losses when there is a downturn in the economy. The portfolio losses lead investors to reassess the value of the franchise, often leading to a fall in the bank's value that is a multiple of the realized portfolio loss.

Real franchise value cannot be produced simply by taking risk. It can only be produced by taking risks that are worth more than they cost the bank to hold or distribute. The path to franchise profitability lies, first, in understanding the market value of loans at the time of origination.

## Valuing the Loan: Internal Versus External Benchmarks

In applying RAROC analysis to a loan origination, inside the RAROC calculation, a loan valuation is being performed. In theory, this loan valuation should yield the same result that one should get by determining the external secondary market value of a comparably risky loan. In practice, there is usually little relationship between these two values.

The following table illustrates the two methods.

Let  $L$ =libor  $S$  = contractual loan spread  $k$ =required capital

$HR$  = equity hurdle rate  $pr(D)$  = probability of default

$PE$  = portfolio effect (return for undiversified risk)

- External Market

Spread at origination =  $L + S$   
 Required return = riskfree rate  
 + expected loss premium  
 + risk premium  
 + portfolio effect

Riskfree rate =  $L$   
 $ELP = pr(D) * LGD$   
 $RP =$  additional return required  
 by comparable risk assets in  
 market  
 $PE = RP * (\text{undiversified}$   
 $\text{portfolio risk}) / (\text{systemic}$   
 $\text{portfolio risk})$

Skim to origination =  $L + S - [L +$   
 $ELP + RP + PE] =$   
 $S - ELP - RP - PE$

- Internal Bank

Spread at origination =  $L + S$   
 Cost of debt funds =  $(1-k) * L$   
 Cost of equity funds =  $k * HR$   
 Hurdle rate = riskfree +  $\beta_E * ERP$   
 riskfree rate =  $L$   
 $ERP =$  equity risk premium  
 $CC = pr(D) * LGD$

Skim to origination =  $L + S - [(1-$   
 $k) * L + k * (L + \beta_E * ERP) + CC]$   
 $= S - k * \beta_E * ERP - CC$

Note: $CC = ELP$ $k * \beta_E * ERP$ should equal $RP + PE$
--

The external market method says that the required return on the loan should be equal to four components: the risk free base rate, the expected loss premium, the risk premium, and any additional return required to compensate for under-diversification in the portfolio. We take LIBOR as the risk free base rate, and set the expected loss premium equal to the actuarial expected loss on the loan by multiplying its default probability times its loss given default. The risk premium is calculated by looking at the additional return required in the market for comparable risk assets, and the portfolio effect scales up the risk premium by the ratio of the undiversified to the undiversifiable risk of the asset, as determined by a loan portfolio model.

The revenue or "skim" to origination is the difference between the contractual spread ( $S$ ) and the required return, or  $S - RP - PE - ELP$ .

The internal bank method focuses on the cost of producing the loan. Using portfolio analytic methods, the equity fraction of the loan's financing is determined to be  $k$ . The cost of producing the loan is thus  $(1-k)$  times the cost of debt funding (assumed to be LIBOR), plus  $k$  times the cost of equity funding, which is assumed to be the required equity hurdle rate ( $HR$ ). The required equity hurdle rate is usually based upon an analysis of the required market return of the bank's equity, using CAPM. In the analysis, the required market return is the risk free base rate (LIBOR) plus the bank's market sensitivity (its beta,  $\beta_E$ ) times the equity market risk premium ( $ERP$ ), which is usually assumed to be 6%-8% based upon long term averages. Finally, the "credit cost" ( $CC$ ) must be factored in, to cover the actuarial risk of default of the loan. This is calculated as the probability of default times the loss should default occur.

Under the internal bank method analysis, the "cost" of producing the loan, based upon these funding costs, reduces to  $L + k * \beta_E * ERP + CC$ . The gross economic profit on the loan is

determined by subtracting this cost from the loan return, equal to LIBOR plus the contractual spread (S). The result is  $S - k \cdot \beta_E \cdot ERP - CC$ .

The gross economic profit under the external market approach was shown to be  $S - RP - PE - ELP$ . The measures under the two approaches should be identical if they are done correctly; gross economic profit is an objective concept. The equivalence can be seen by first noting that the bank's credit cost is the same as the market's expected loss premium, i.e.  $CC = ELP$ . Second, the bank's assignment of capital to the loan,  $k$ , should account for the risk of the loan in the portfolio, i.e. it should include both the *undiversifiable* risk of the loan plus its *undiversified* risk. The use of the loan's capitalization rate ( $k$ ) times the bank's equity beta and assumed equity pricing is an alternative (approximate) approach to obtaining the debt market's required premium on the loan, deriving it from the equity market's required premium.

In other words, if done properly, the risk premium plus portfolio effect from the debt market should equal the capital ( $k$ ) times the required risk premium on the bank's own equity ( $\beta_E \cdot ERP$ ). For example, a one-year A-quality loan with a 10bp EDF value might require about 1.5% capital. The bank's required equity premium, above the risk free base rate, is probably about 8%. Thus the A loan would require about a 12bp risk premium ( $= .015 \cdot .08$ ) plus an expected loss premium of 4bp ( $= (.001 \cdot .4)$ ), or an overall spread of 16bp.

In practice, the numbers coming out of most banks' RAROC models do not even approximately measure the real value or required spread on a loan. Because of the inclusion of costs and cost allocations, capital for market and operating risks, arbitrary determination of equity hurdle rates, and so forth, most models yield artificial numbers. These numbers have also usually been extensively "massaged" to satisfy internal political purposes.

However, while one could spend much time identifying the distortions present in any given RAROC approach, it is more productive to simply recognize that the external market approach provides a better and more straightforward method for determining franchise profitability.

The external market approach can be characterized as "marking to market" the loan book. There are few issues in banking that create as much controversy as "mark to market" for the loan book. The issues range from the theoretical foundations of bank accounting to the technical feasibility of marking lightly traded or untraded assets to market. The following section attempts to address these issues.

## Marking the Credit Book to Market

There are two primary reasons why one "marks to market" in finance. The case of credit has important differences with both of these. In evaluating the need and the ability to mark to market, it is critical to understand the context and the purpose of the exercise.

Banks and non-banks mark their trading books to market as part of the risk control and management of traders. Traders are generally supposed to make money by buying at one price and selling at a different one. Marking to market is a key discipline in correctly measuring trader profits, avoiding the warehousing of losing trades at artificial prices, and assessing accurately the losses or gains in desk inventory in a fast moving market. In this setting, where positions are supposed to be liquid, there is particular disdain for using "model" rather than actual market prices, because such prices are viewed as being particular subject to abuse by unscrupulous traders. For instance, some firms have instituted practices of marking down all positions without good external marks if they have been held for longer than some set number of days.

A different mark to market setting is that associated with mutual funds. Portfolio purchases and sales are supposed to be transacted at the market price to avoid a subsidy or loss to other owners of the fund. Here there is much less necessity to get a true market price on every asset, because what is important is the total value of the portfolio. Errors on individual prices tend to cancel against each other. Pricing services that provide prices for mutual funds know this well, and often rely upon modeled prices when no current external price exists. For instance, in the bond market, most prices on any given day are modeled or indicative, rather than based upon actual transactions. Nonetheless, this provides an adequate mechanism for fund pricing, where it would not necessarily suffice for trading desk management.

The case of bank credit portfolios is distinct from these cases in several important ways. First, the credit portfolio is generally held to term, so that the trading desk risk management objective of avoiding mispriced inventory is not paramount. Second, the entire credit portfolio is not being sold, so the goal is not to value the entire portfolio for transaction purposes.

What, then, is the purpose? In the current context, the answer is to measure correctly the impact, ultimately on shareholders, of decisions to originate, hold or sell individual credits. When the real economic effect of decisions is correctly measured, it can be used to incent the behavior of the individual decision makers within the bank. The first section of this paper was a detailed presentation of the nature of the problem and the measurements and incentives needed to address it.

Notice that the proposed use here of mark to market for bank loan portfolios is directed internally to management, not externally to support investor decisions. While there may be good arguments for external disclosure of portfolio values, that is beyond the scope of this paper, and moreover, has no necessary or logical link with the use of mark to market for internal management purposes.

The argument that is most often sounded against mark to market in the credit book is that loans are illiquid, there are few good secondary market prices, and thus all the prices would be "made up" (i.e., modeled). This may be a damning argument in the trading book, but in the loan book it is not. Why not? First, because the alternative right now is accounting prices. In other words, everything is worth what it was originated for, unless it has become "impaired" by accounting standards. These are the worst "model" prices that exist.

Second, because banks have already recognized that accounting prices create bad incentives, they have been constructing alternatives going back almost two decades. The previous section was an analysis of the most common approach, namely RAROC measures.

Although the idea of RAROC is a well-motivated response to the need for better economic measures than accounting prices, RAROC prices invariably still contain major distortions, mostly due to their own grounding in internal accounting measures rather than external market measures.

Third, by using existing data and analysis, we can convert external market data into better economic prices for loans than we can achieve via accounting or RAROC models. Where real secondary market prices for loans exist, we should use them, but we should not disdain using modeled prices, because the alternatives are all uniformly worse. By specifying the objective as conformity with secondary market prices, we create a clear standard by which such prices can be evaluated and improved. No such standard exists for accounting prices or RAROC models.

Whether such an approach should be called “mark to market” is a semantic argument. In the bond market, where true market prices are rare, it is called that. In the derivatives market, where the standard is not secondary market prices, it is called that. The crux of “mark to market” is not that every price be an actual secondary market transaction price, but that the standard is conformity to such prices when they are known. The real issue lies in understanding the purpose and context of the use.

## Conclusion

Banks can be thought of as comprising two parts: a portfolio, which contains all the financial assets (and contra-assets) originated by the bank; and the franchise, comprising the remaining non-portfolio activities of the bank. When we make this division, and focus on market rather than book value, we make a number of interesting discoveries.

- Most large bank franchises have market values in the range of \$30 billion to \$100 billion; they are very large service businesses.
- The risk in the franchise is much higher than the risk of the portfolio, but it is similar to that of other business services firms of the same size.
- The risk and return characteristics of the franchise cannot be readily determined by a detailed consideration of its components; it can generally only be measured in its totality.
- Similarly, the effect on value and risk of adding or subtracting component parts of the franchise is difficult to determine because of the complex interactions between business components, scale economies and strategic positioning.

When we isolate out the portfolio, we note that the portfolio has the following characteristics:

- A very low level of risk, with the value generally fluctuating within a 2% band.
- Mostly downside risk; there is very little opportunity to produce upside in the portfolio
- The risk of the whole can be readily constructed from the risk of the parts; the effect of adding or subtracting individual assets on risk and return can be readily measured.

A number of implications flow from these observations. For instance, the very low risk of the portfolio means that we can finance the portfolio with very high levels of leverage (>90%), whereas the high risk of the franchise means that it can only support low levels of leverage if it is to have satisfactory credit quality.

The major point, however, is that the upside potential of the bank resides in the franchise, not the portfolio. Thus from the standpoint of managing the bank to produce value for shareholders, portfolio management is largely defensive. It is a steady stream of earnings from the franchise that is highly valued by stockholders.

However, in practice, banks have often taken the opposite tack. The false appearance of a steady stream of earnings can be produced by a bank that is simply taking more portfolio risk. Eventually, such risks end up producing significant portfolio losses when there is a downturn in the economy. The portfolio losses lead investors to reassess the value of the franchise, resulting in a fall in the bank's value that is a *multiple* of the realized portfolio loss.

Real franchise value cannot be produced simply by taking risk. It can only be produced by taking risks that are worth more than they cost the bank to hold. Franchise profitability depends on intermediating more efficiently, and thus more profitably, than the competition, and seizing market share as a result. The key to understanding profitability lies in understanding the market value of an underwriting at the time it is originated.

Banks, trying to determine profitability, have relied upon RAROC models. This development of the 1990's was, in fact, a big improvement over simply focusing upon book accounting. Unfortunately, the numbers coming out of most banks' RAROC models do not even approximately measure the real value or required spread on a loan. Because of the inclusion of costs and cost allocations, capital for market and operating risks, arbitrary determination of equity hurdle rates, and so forth, most models yield artificial numbers. These numbers have also usually been extensively "massaged", sometimes to satisfy internal politics, often simply to make them more consistent with reality.

With the development of traded credit markets, it is now feasible to produce simpler and more direct estimates of loan market values. While such measures are critical to loan portfolio management, they are even more important to understanding the profitability of the bank's franchise. In short, the purpose is to correctly measure the impact on shareholder value of decisions to originate, hold or sell individual credits. Market value measurement

- facilitates active portfolio management
- improves daily decision making around asset origination

- correctly determines the profitability of business units for tactical business purposes
- better informs strategic decisions around the franchise.

The next generation of bank profitability modeling will move from a focus on portfolio risk and capital to a better understanding of the effect of strategic and tactical decisions on franchise value. The perspective of portfolio and franchise, and the developments of quantitative credit risk measurement, will be key components of this approach. Ultimately, it is the expansion of traded credit markets that creates the opportunities for innovations in intermediation, as well as generating the secondary market values that are key to understanding and unlocking these opportunities.

## Appendix

The objective of this appendix is to describe a practical methodology for determining the market value of bank loans and other credit sensitive instruments. Market values in the first instance come from markets. However, the non-continuous nature of credit markets means that there are not actual secondary market prices available for most instruments at any given moment. This is not a fatal flaw; many markets have similar characteristics.

All of a corporation's liabilities, including its equity, are derivatives of the same underlying firm. This means that there are definite relationships that exist between the values of these liabilities. By exploiting these relationships one can obtain a good estimate of secondary market value for a given liability from the liabilities with known market values, including the equity.

The appendix is broken into two sections, the first addressing straight credit risk, and the second describing the valuation of options that are embedded in most loan agreements.

### Valuing Credit Risk

In valuing a loan, understanding its credit risk is paramount. The reason is straightforward: as a floating rate instrument, the loan has little or no interest rate risk or other so-called "market" risks. If it were simply a bullet loan, with no hope of recovery upon default, we could value it solely by reference to its default probability over its term, given knowledge of how the market place was valuing credit risk.

This point elicits the second primary feature of loan valuation, namely the loss should the loan default, known as the "loss given default" or LGD. Based upon the historical experiences of banks and bond investors, we have some idea of what values are reasonable to assume for different types of credit instruments. We are just at the beginning of a long research program that will hopefully lead to a better understanding of the market value impact of all aspects of the loan agreement or bond covenant. One of the advances described here is the isolation and measurement of the "market" LGD assumptions that are embedded in actual bond prices, as a function of seniority, structure, and industry.

We begin with the valuation of pure default risk. Option pricing theory<sup>1</sup> provides a relatively simple characterization of the relationship between the default probability for a firm and the valuation of its liabilities. Prior to options pricing theory, the approach would have been to discount the expected cash flows of a loan at a discount rate appropriate for their systematic risk. However, there was no consistent method for determining the systematic risk of loan cash flows.

Black and Scholes<sup>2</sup> showed that the correct approach was not to discount the cash flows at some risk-adjusted rate, but rather to directly adjust the value of the cash flows for their

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<sup>1</sup> See Black and Scholes[1973], Merton[1973,1974], Black and Cox[1976], Ingersoll[1977a,b], Vasicek[1984,1998]

<sup>2</sup> Black and Scholes[1973]

systematic risk. They also showed that this adjustment could be made by a specific modification of the default probabilities of the cash flows.

Consider the case of a single cash flow,  $F$ , due at a single future date,  $t$ . Let  $r$  be the continuous discount rate to time  $t$  for a default-risk-free cash flow<sup>3</sup>. Then the value of the cash flow today is given by<sup>4</sup>

$$V = F \cdot \exp(-r \cdot t) \cdot (1 - q \cdot \text{LGD}),$$

where  $q$  is the cumulative default probability to time  $t$  (adjusted for systematic risk per Black-Scholes), and LGD is the expected percentage loss should the borrower default. The relationship between the risk adjusted cumulative default probability,  $q$ , and the actual cumulative default probability,  $p$ , is given by<sup>5</sup>

$$q = N\left[N^{-1}[p] + ((\mu - r) / \sigma) \cdot t^{1/2}\right]$$

where  $\mu$  = the instantaneous expected return to the borrower's business

$\sigma$  = the volatility of the market value of the borrower's business, and

$p$  = the cumulative default probability to  $t$ .

Intuitively, the cash flow is valued as if the default probability were  $q$ , which is greater than the actual probability  $p$ . The actual default probability  $p$  can be converted into the risk adjusted probability  $q$ , based on the formula above, which uses information about the risk and return characteristics of the borrower's total business.

The difference between the actual and the risk-adjusted probability is due to the systematic component of the default risk. The difference is based upon the expected return that is required for the systematic risk of the borrower's business. If the business had no systematic risk, so that its expected return was equal to the risk free return, then the two probabilities would be identical. Since the systematic risk of the business is communicated to the cash flow via default, the amount of risk premium required for pricing the default risk is directly related to the amount of risk premium required for the business. The risk adjustment to the default probability is exactly the risk premium required to compensate for the systematic risk of default of the cash flow.

An extension of the valuation formula for a single cash flow to the case of multiple cash flows is:

$$V = \sum C(t) \cdot \exp[-r(t) \cdot t] \cdot [1 - q(t) \cdot \text{LGD}],$$

<sup>3</sup> See Collin-Dufresne and Solnik[2001] for a discussion of the appropriate risk free rate.

<sup>4</sup> See Bohn[2000a]

<sup>5</sup> Vasicek[2001]

where  $C(t)$  is the cash flow at time  $t$ , and the summation is over all cash flow dates. This formula can be used to value the multiple cash flows encountered in credit sensitive instruments such as loans and corporate bonds. However, to use the formula, we need to empirically determine certain parameter values that are required to risk adjust the default probabilities.

If we assume that the risk premium ( $\mu-r$ ) is determined as per CAPM, we can write

$$\mu-r = \text{beta} * \text{market risk premium, or}$$

$$\mu-r = [\text{COV}(r_A, r_M) / \text{var}(r_M)] * [\mu_M - r] .$$

Thus the expression above in the risk adjustment equation can be rewritten as

$$(\mu-r) / \sigma_A = [\text{COV}(r_A, r_M) / (\sigma_M * \sigma_A)] * [(\mu_M - r) / \sigma_M] , \text{ or}$$

$$(\mu-r) / \sigma_A = \text{corr}(r_A, r_M) * (\text{market Sharpe ratio}) ,$$

where  $r_A$  = asset return

$r_M$  = market return

$\mu_M$  = expected return

$\sigma_A$  = standard deviation of asset return

$\sigma_M$  = standard deviation of the market return.

Substituting this back into the risk adjustment equation gives

$$q = N[N^{-1}[p] + \rho * \lambda * t^{1/2}] ,$$

$$\text{where } \lambda = (\mu_M - r) / \sigma_M$$

$$\rho = \text{corr}(r_A, r_M) .$$

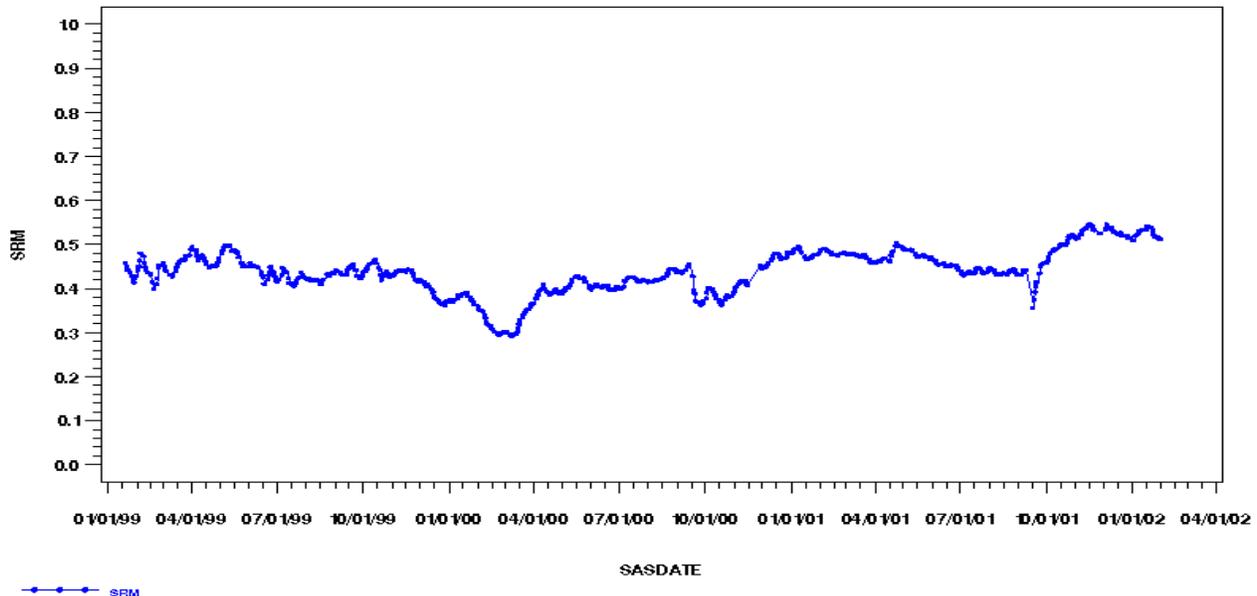
The risk premium on the systemic risk divided by its standard deviation can be interpreted as the Sharpe ratio of the market, or equivalent, the market price of risk. This form of the equation shows that the risk adjustment to the probability depends upon how systematic risk is being priced in the overall market, as reflected in the market Sharpe ratio term, as well as the amount of systematic risk in the borrower's business, as represented by the correlation of that business's return to the overall market.

If one uses the equity market as a benchmark, one expects a value for the numerator of the market Sharpe ratio of about .06 to .08, and for the denominator

of about 0.12 to 0.20. This suggests a value for the market price of risk in the range of 0.3 to 0.7. However, there is no reason to believe this value could not vary through time and

explain some of the variation that one observes in spreads on debt. To implement the model, we empirically estimate the Sharpe ratio for the overall market ("SRM"), from debt market prices<sup>6</sup>.

### SRM estimation

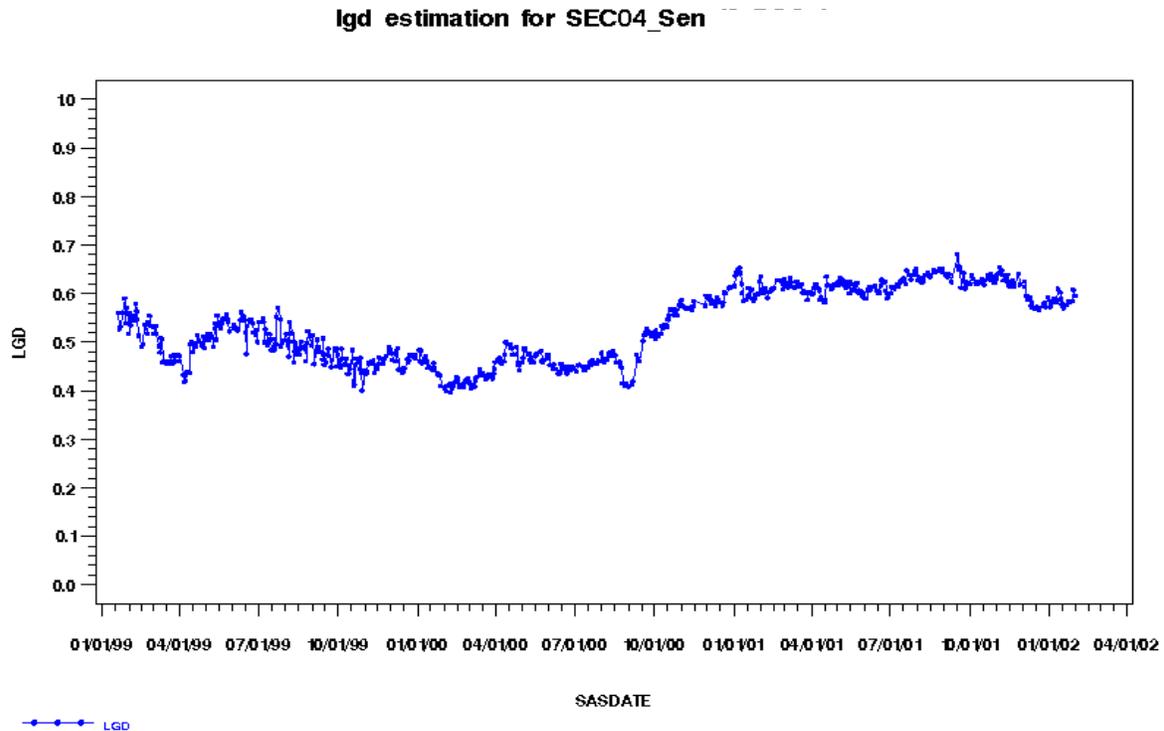


The resulting estimates are consistent with our prior expectations, generally varying around 0.4 to 0.5 in value. Even more striking is the relative constancy of the estimates. As some of the observed variation is simply noise, due to estimation error, the data suggest that risk premium per unit of risk is relatively stable. This means that the large moves over time in actual bond spreads are not due to changes in the pricing of risk, but rather to changes in the underlying level of risk itself.

As noted above we also need a specification for loss given default (LGD). Utilizing historical averages of debt values subsequent to default provides satisfactory aggregate results. However, when we look at individual issues, we find that there are deviations between modeled spreads and actual spreads that tend to be quite persistent. One way to deal with such deviations is to assume that they are related to the industrial sector of the issuer, and to determine LGD as the average model residual across the sector.

<sup>6</sup> See Bohn[2000b] for an earlier version of this work.

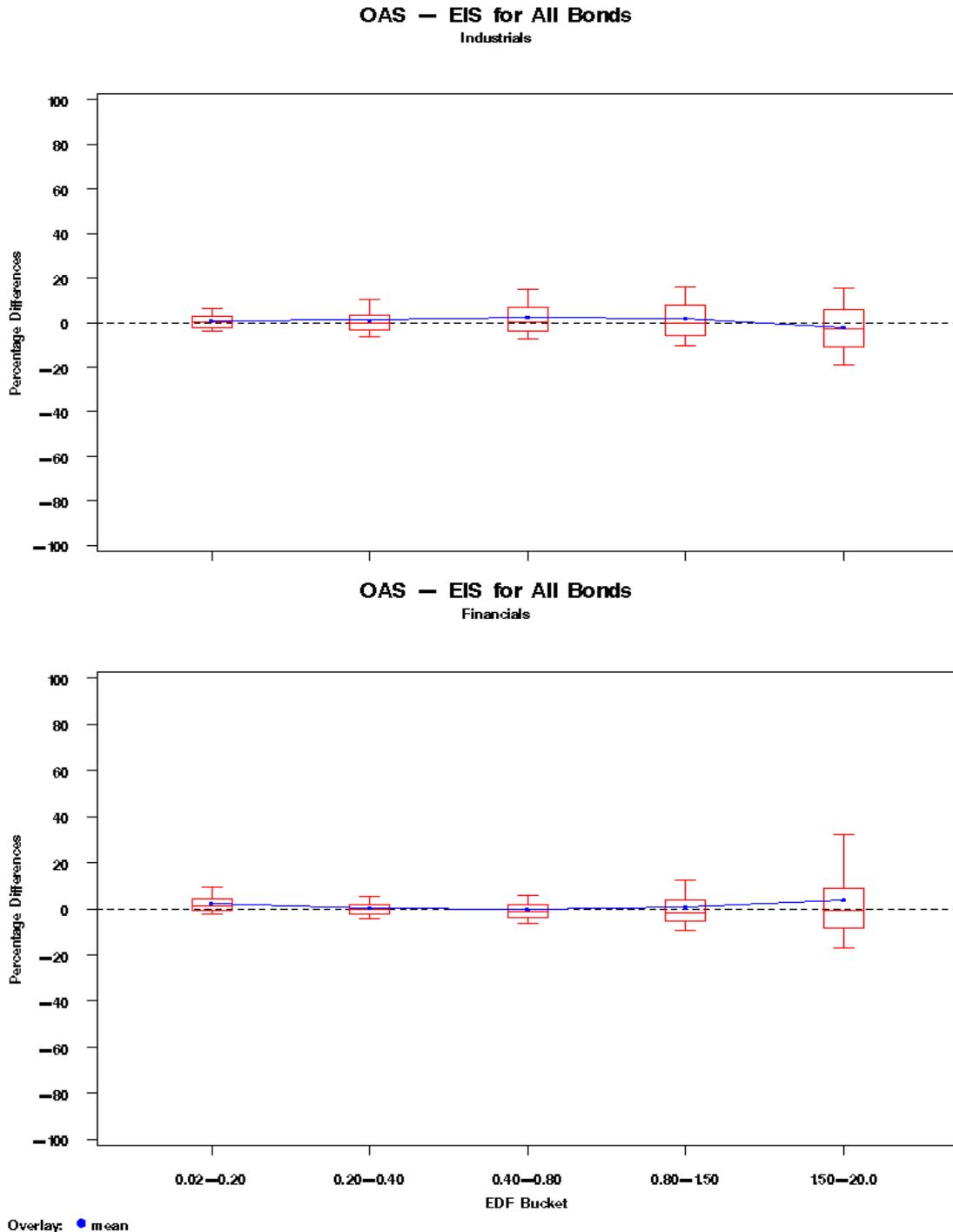
This approach yields plausible LGD estimates that do exhibit some variation over time. An illustrative case is shown in the graph below.



These estimates are obtained by finding the LGD estimate that gives the best fit at a particular date for all senior, unsecured bonds of issuers in the consumer goods sector.

The estimates vary over time, with values predominantly around 0.5, which is consistent with recovery data on senior unsecured bonds in general. The higher values, around 0.6, are from the latter half of 2001, which corresponds to the trough of the recent downturn. It is plausible that recoveries would be lower at such a time, and thus LGD higher. The estimates change in a fairly smooth way through time, suggesting that the most recent estimates would be quite useful in predicting the immediate subsequent period's value.

Looking across a diverse group of bonds, the variation in expected default frequency explains about 70% of the variation in bond prices. Once we also account for sector differences in LGD and seniority, the modeled prices account for about 90% of the variation in bond prices. The following charts show the range of observed deviation between reported and modeled bond prices, by quality range.

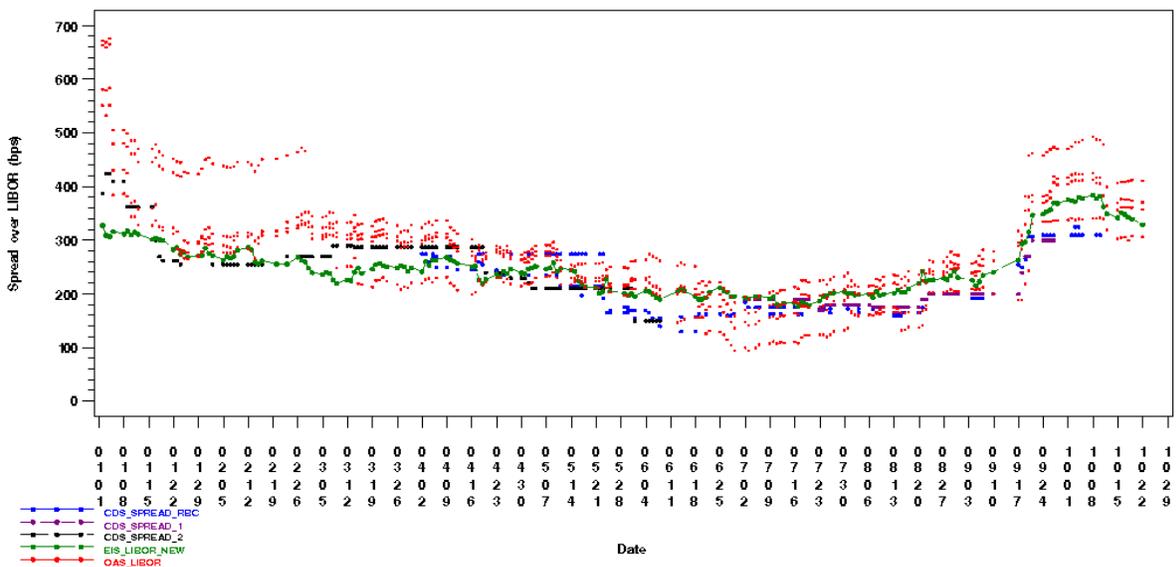


The graphs illustrate the quality of the fits in both the industrial and financial sectors. The red boxes show the interquartile range of errors, which vary from a few basis points for investment grade issues, to about ten basis points for subinvestment grade issues. These represent the median errors; half of all issues have smaller errors. For the issues with larger errors, the endpoints of the lines represent the 10<sup>th</sup> and 90<sup>th</sup> percentiles, respectively. These are also quite narrow.

Looking at both charts together, there are no significant differences in the ability to fit bonds of industrial versus financial issuers.

What do these results mean at the individual issue level? The following graph shows a representative example, using the spreads on various instruments for Goodyear Tire and Rubber. The graph shows data from 2001. It contrasts three spreads: the model spread (EDF implied spread or EIS), the option adjusted bond spread (OAS) from the secondary bond market, and the credit derivative swap premium (CDS). One expects the option adjusted spread and the credit derivative swap premium to be near each other, because there is an arbitrage between the two if the prices are different. However, the model spread should only be close if (i) the expected default frequency is correctly measured and (ii) the model specification is appropriate.

**EDF-implied Spread vs CDS Spread vs Bond Spread from 20010102 to 20010928 for GOODYEAR TIRE & RUBBER CO**



The model spread is shown in green, multiple credit derivative swap quotes are shown as blue or purple bars, and option adjusted spreads on different bond issues are shown in red. Note that there is good correspondence, as expected, between the bond and credit derivative spreads (except for a small number of issues that had different seniority provisions). The main point is that the model spreads also do an excellent job of tracking the bond and credit derivative pricing. The deviations between the model pricing and the bond pricing are of approximately the same magnitude as that between the bonds and the credit derivatives, where a direct arbitrage exists.

Overall, the aggregate results provide strong evidence that the valuation of credit sensitive instruments can be accurately determined using information from the stock, bond and credit derivative markets. The valuations are not only good on average, but they are good for individual instruments for different dates, thus creating better pricing guidelines for credit decisions than can be obtained via alternative methods, such as RAROC models.

## Evaluating Optionality and Other Loan Features

In the previous discussions, the primary feature of the loan was taken to be its credit risk. There are other aspects of loans that need to be taken into account in valuation that can be loosely characterized as optionality and fee structures.

The nature of almost all contracts is that they address contingencies and thus create options. Loan agreements are no exception. Almost all bank facilities can be paid off early, at par, at the option of the borrower. This creates a situation where a borrower can repay the facility if its credit quality improves, or can use the fact it could repay to obtain better terms on the loan. This option is clearly valuable to the borrower.

Another important but different loan option is the right to draw and repay at will that is explicit in a revolving credit agreement. The borrower can choose how much to borrow, any time during the life of the revolver, subject to satisfying certain conditions.

In addition to these two principal options, there are also special fee structures on loans (and on some corporate bonds). A good example is so-called "grid pricing", where the fees charged on the bank facility vary as a function of some variable, such as an accounting ratio or an agency debt rating.

Over the last thirty years, there has been a tremendous advance in evaluating financial options, beginning with the work of Black and Scholes. As a result, there are technical methods available for the evaluation of almost any type of option. However, before proceeding into the technical details of option evaluation, there are two key points that need to be made. The first is to distinguish a primary option from a secondary option. If you are buying a stock option, the purpose is clear, namely to exercise the option when it is in your financial advantage to do so. The option has no other purpose. It is a pure option.

Most options in contracts are secondary options. They represent mechanisms for dealing with unspecified contingencies. For instance, the pre-payment option in a home mortgage is designed to deal with an unanticipated home sale. Once a secondary option is created, it may evoke the same type of behavior as a primary option. For instance, homeowners are flooded with refinancing proposals whenever mortgage rates fall, independent of any home sale. On the other hand, there may be a variety of reasons why such secondary options are never exercised like pure options. Mortgage prepayments are decidedly difficult to model and are not consistent with the exercise of pure options. In valuing secondary options, it is necessary to look at the empirical pattern of exercise in order to achieve good results. Simply relying upon a conceptual model of options exercise can be quite misleading.

Second, option valuation approaches are based upon modeling the distribution of the underlying asset that is driving the options value. For instance, in the case of stock options, one makes specific assumptions about the distribution of stock returns. In the case of bond prepayments, it is the distribution of the term structure of interest rates that must be modeled. When one turns to valuing the options in loans, because they are floating rate instruments, the underlying process driving the option value is the credit risk. There has been little empirical work done on modeling credit risk distributions. Almost all the

academic work to date on credit risk has either been highly conceptual, such as that of Merton, or it has used questionable empirical shortcuts. The three leading shortcuts are:

- assume that agency debt ratings are an accurate, timely and complete picture of the underlying credit risk, and model ratings behavior as a substitute for credit risk;
- assume that default is a truly exogeneous event that can occur to any borrower at any time without warning; or
- assume that credit risk behaves like interest rate risk and can be readily measured from the volatility of bond prices.

It is not difficult to show that all of these assumptions are highly counterfactual.<sup>7</sup> The result is a large and increasing number of models for addressing optionality in credit instruments that cannot be satisfactorily implemented empirically, but that look good on paper. Thus the first step in evaluating loan optionality is to address the underlying process determining the firm's credit risk from an empirical standpoint.

The research work at KMV<sup>8</sup> over the last twelve years has established the following points:

- Default is not an unanticipatable, exogeneous event, but is related to the firm's *ability* to pay its bills.
- The firm's ability to pay its bills is directly related to how much the firm is worth relative to what it owes. If it is worth much more than it owes, then it has little likelihood of default; if it is worth less than what it owes, it is very likely to default. By monitoring its value relative to its liabilities, its current likelihood of default can be measured quite accurately.

The latter point is summarized in a measure called the "distance to default" (DD). It is defined for a particular future horizon, such as one year or five years. It measures the distance between the firm's market value and the level to which the value would have to fall for the firm to default (default point), standardized to the volatility of the firm. For instance, if a firm's value had to fall 80% to bring it to its default point, and if the standard deviation of the firm's value was 20% per year, then its distance to default would be four standard deviations. If its standard deviation were 10% per year, then it would be eight standard deviations. A firm that is four standard deviations from default is about BB-quality; a firm that is eight standard deviations from default is about AA-quality.

The one-year distance to default provides a good metric for evaluating how firms' credit qualities change through time. Beginning with a particular initial credit quality (as measured by DD), we can evaluate how firms' credit qualities tend to change through time. The following set of charts shows the cumulative probability distribution of DD values in one year for different horizons for different initial values of DD. Each chart represents a

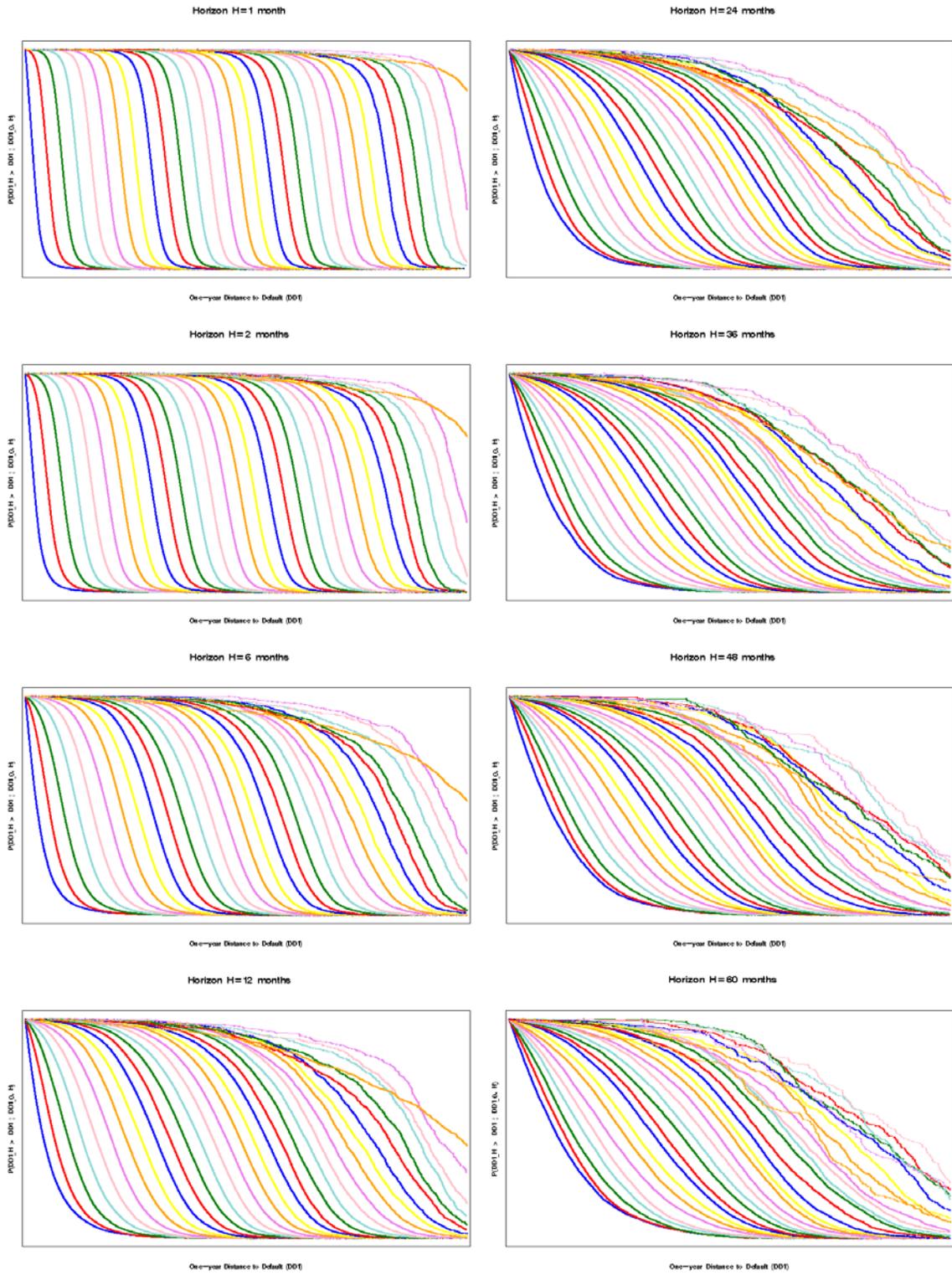
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<sup>7</sup> See Miller [1998] for evidence on ratings and default probabilities. See Kealhofer, Kwok and Wenlong [1998] for evidence on the characteristics of rating transitions processes.

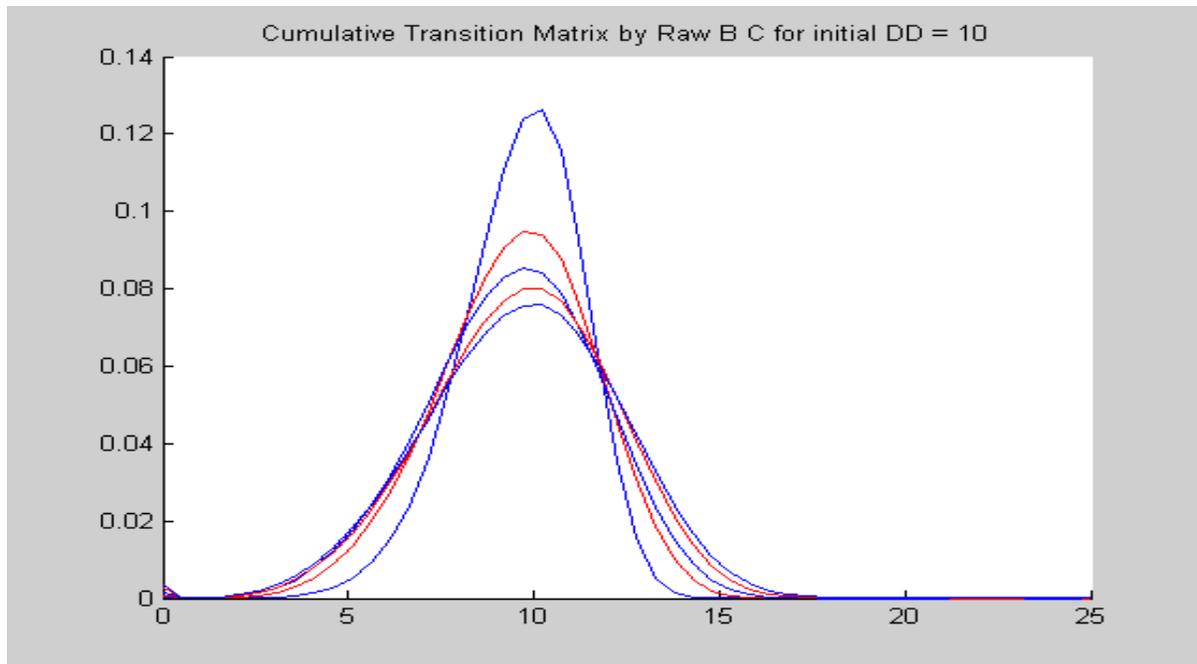
<sup>8</sup> See Kealhofer[2002]

different horizon, going from 1 month to 60 months. On each chart, different lines represent different initial DD values.

Each line on a given chart represents the probability (on the Y axis) of getting a DD value at the horizon that is greater than the corresponding value on the X axis. The distributions summarize the evolution of firms' capital structures, namely the effect of exogenous changes in firm value and liabilities, as well as the firm's choices in managing its liabilities. First, the behavior of firms' capital structures has considerable regularity (the distributions on the far right of the graphs can be ignored as they correspond to extremely high credit quality firms where there are too few observations to obtain reasonable measurements). Second, the shapes of the distributions are interesting; for low initial DD values the distributions are skewed to the right, while for high initial DD values, the distributions are skewed to the left. These skewnesses result from deliberate actions on the part of low quality firms to avoid further deterioration in quality, contrasted with a bias of high quality firms to use additional leverage opportunistically.



Third, there is a tendency towards relatively more extreme outcomes at short horizons than at long horizons, where relative is understood with respect to “normal” outcomes at that horizon. This point can be made more clearly by looking at the following chart. It shows the probability density functions of DD values for horizons of one to five years, for a single initial DD value of ten, in contrast to the cumulative distributions shown in the previous set of charts, where all initial DD values were shown as well. The “height” of the density function gives the likelihood of the subsequent DD value at the horizon (shown on the horizontal axis).



For shorter horizons, the distributions are more compact (vertical); for longer horizons they are more spread out. In essence, more can happen over five years than over six months. In this sense, longer horizons generate more extreme outcomes. However, the shorter horizon distributions have relatively “fatter” tails; there is a greater probability of relatively extreme events given the general tendency. The longer horizons have more “normal” probabilities of extreme events; extreme events are more in proportion with the general risk tendency.

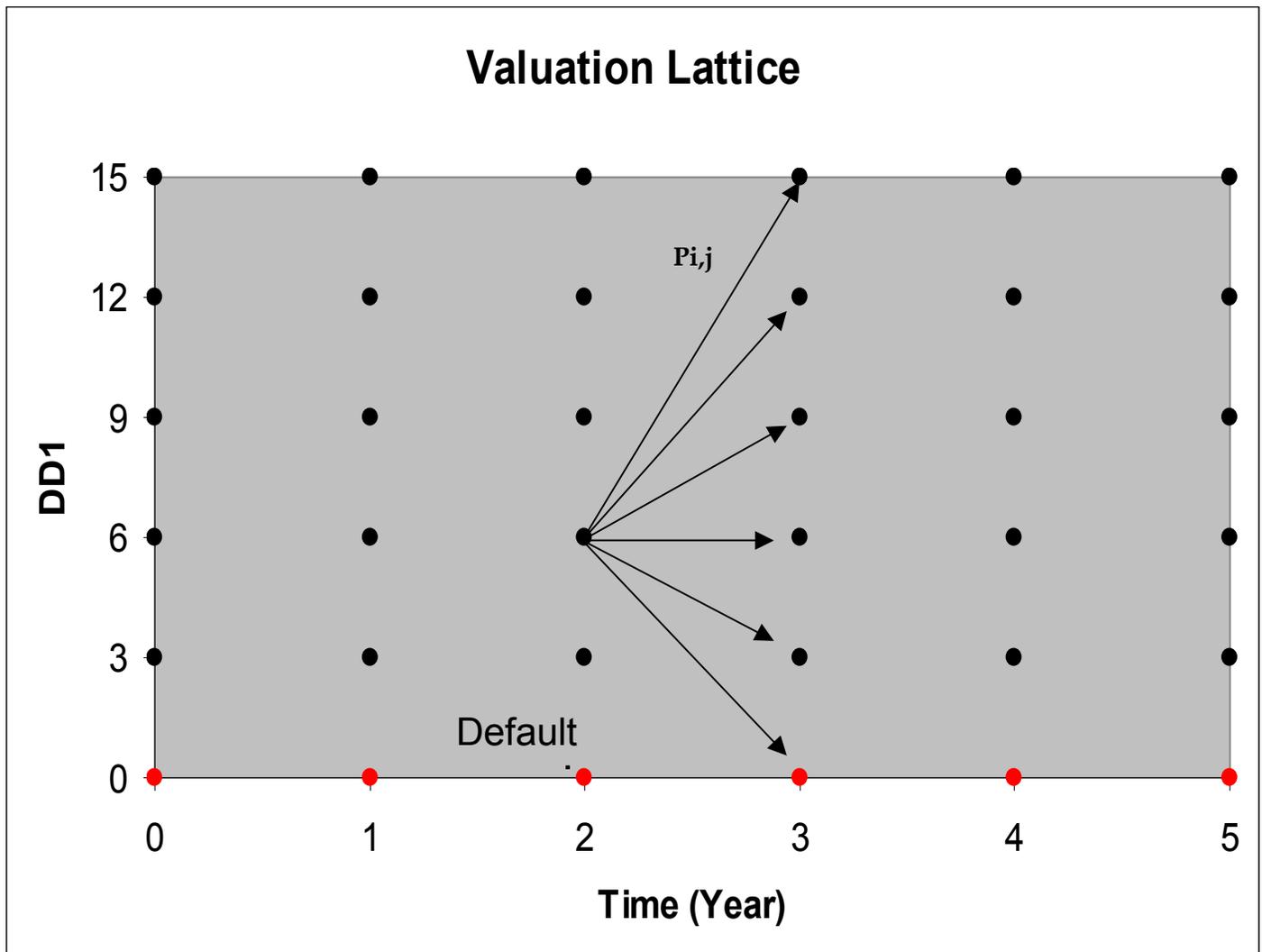
### The Prepayment Option

These distributions provide exactly the information that is needed to evaluate option characteristics that depend upon credit quality. For instance, as discussed above, most banks loans are prepayable without penalty. This option will be in the money when the credit quality of the borrower improves sufficiently relative to the fees charged on the loan. The distribution makes it possible to determine the probability over the life of the loan that the borrower’s credit quality will improve sufficiently that it is optimal for the borrower to repay the loan early.

In practice, the information in the continuous functions shown above is approximated by allowing for a finite set of outcomes at a finite set of dates. For each outcome, there is a

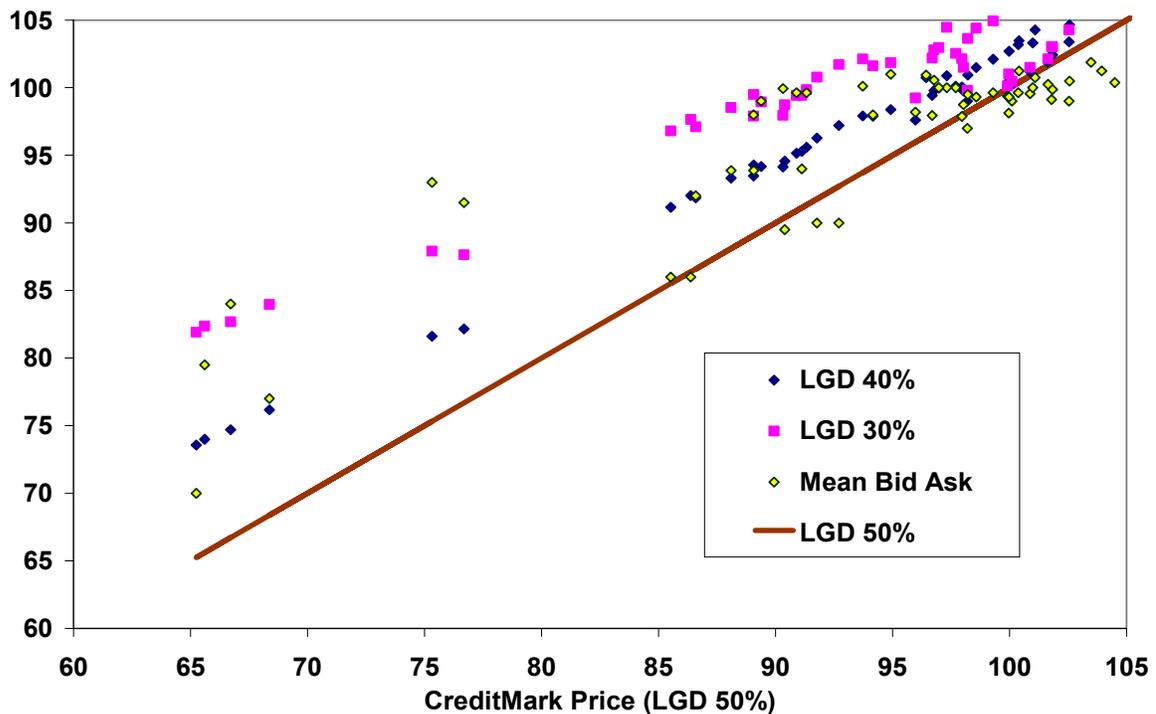
probability of occurrence, as well as a probability of “transitioning” to each of the possible outcomes at the next date. These probabilities and outcomes can be used to value the instrument, as well as to determine the “optimal” (i.e. value maximizing) actions of the borrower in exercising various options.

The following graphic represents this process.

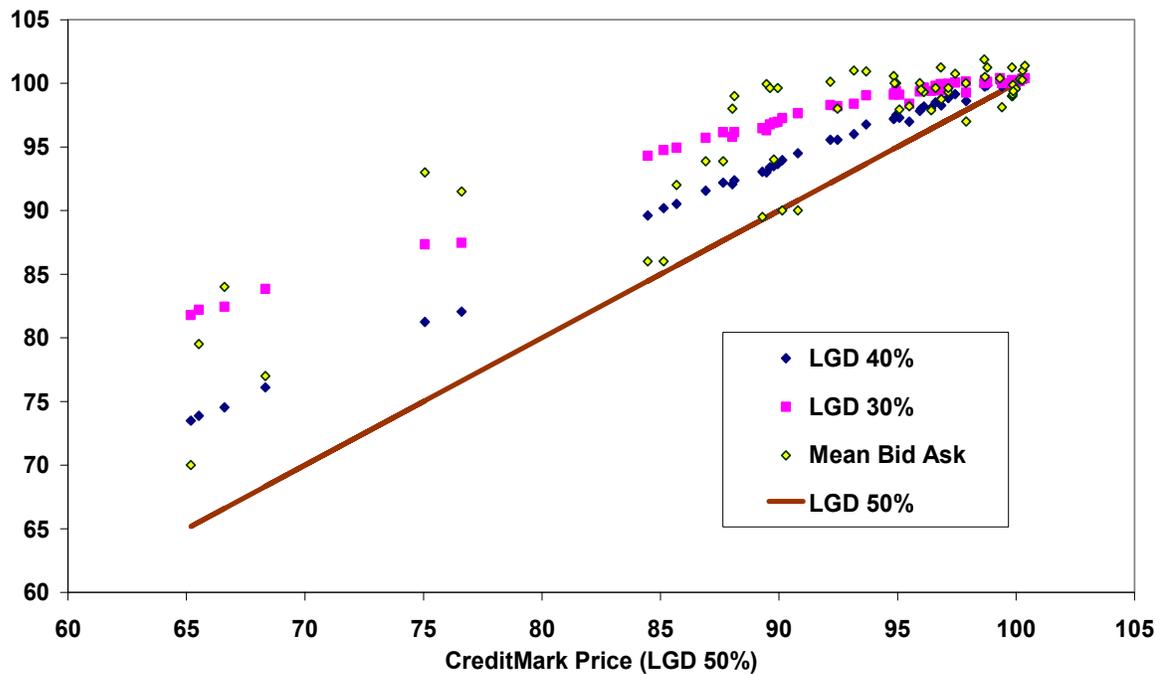


The following two charts provide empirical evidence on the valuation of prepayment options using this approach. Pricing data on 57 loans was compared with model values, first, without accounting for prepayment, and second, with the prepayment option valued as described above. In this exercise, no attempt has been made to introduce cost of exercise, so the prepayment options are valued as primary options. Three different model values are computed, based upon LGD assumptions of 30%, 40% and 50%.

### CreditMark Valuation With Prepayment Option Turned Off



## CreditMark Valuation With Prepayment Option



The contrast between the model values in the two graphs indicates the theoretical effect of prepayment. For loans that are not near par, there is little or no effect on the values. For loans at or near par, the presence of the prepayment option reduces the value of the loan. The actual prices, based upon the mean of the bid-ask price and shown on both charts as yellow diamonds, strongly support a prepayment valuation effect of this type. Once one accounts for prepayment, it is also evident that most of the loans are valued consistently with LGDs of about 30%, which is reasonable, based upon reported recoveries on defaulted senior bank debt.

There are also a smaller set of loans whose prices indicate expected recoveries that are significantly less, with LGDs of 40%-50%, which are closer to the historical norms for senior, unsecured public bonds. This variation in LGD is consistent with what we have observed previously in public bond prices, and underlines the importance of using issue specific estimates of LGD.

Finally, the pricing data also provides evidence that valuing the prepayment option as a primary option overstates the value of the prepayment option to the borrower, and thus understates the value of the loan. This can be seen by the tendency of the model values in the second chart, assuming 30% LGD, to underestimate the actual loan prices for loans near par. Using this data, the implied costs of exercise can be determined, and used with the model to ensure that the prepayment options are not overvalued.

### The Usage Option

The other major option that must be addressed in loans is the “usage” option in a “committed” revolving credit agreement. Under a revolving credit agreement, the borrower

can borrow up to the full amount of the commitment at any time prior to the maturity of the agreement.

It is difficult to construct a “theory” of the usage option. If one assumes that the right to draw against the facility is not restricted in any way, and that there are no transactions costs, then there is no necessary reason for the borrower to draw the facility until a moment before default. This is approximately the circumstance of drawings against back-up lines for commercial paper issuers. The back-up line is used as an implicit (or explicit) guarantee that the commercial paper will be repaid, and the borrower then raises funds at approximately the risk free rate given the guarantee.

In practice, for many revolving credit facilities, the commitment to lend is conditional, which means that the revolver cannot be used with third parties as a guarantee. This means that if the borrower needs to raise funds, they may be borrowed under the revolver. However, the amount that the borrower will borrow against the facility depends upon both the cash flow needs of the borrower, and the cost and availability of other forms of financing. Although it is not unreasonable to assume that on average the amount borrowed under the revolver will increase monotonically as the borrower’s credit quality decreases, there is no necessary reason for this relationship to exist for any individual borrower.

The only theoretically obvious condition is that somewhere near default, the borrower will be incented to borrow as much as possible. Default means that one cannot pay ones bills, whereas an unused bank line means that there is unused borrowing capacity that could be used to pay bills. Since the equity holders of a firm will want to keep their equity claim “alive”, they will use any borrowing capacity they have to pay bills rather than to default.

Again, in practice, there is evidence that firms in default do not necessarily borrow the full amount of their commitments. One conjectures that this is due to several possibilities. Borrowers have to satisfy technical requirements in order to draw against the facility. These requirements may make impossible to borrow, for instance, if they are required to provide certain types of collateral in order to draw down the line. Other requirements may simply make it more difficult, such as satisfying certain conditions based upon accounting numbers. Borrowers that anticipate being unable to meet these conditions will be incented to draw sooner, and sometimes due to unanticipated events, will not fully draw their lines before they are unable. Finally, there may be circumstances when a firm intends to re-organize under bankruptcy law and does not want to antagonize a potential future lender by drawing against a line and then promptly defaulting.

We begin by assuming that the maximum amount that will be drawn against the revolving facility is some fraction of the commitment amount. This is called the “usage given default” or UGD, as it will be drawn in the event of default. The first and most critical point in valuing a revolver is that the actual usage has little to do with the risk of the facility: *the risk of the facility is determined by the borrower's probability of default (EDF) and the amount at risk (UGD\*commitment amount)*. The anticipated pattern of future usage is almost solely relevant for determining what fees the revolver will earn. Higher usage generally produces greater fees, so everything else equal, a revolver with higher anticipated usage will be more valuable.

The actual drawings against the revolver can be assumed to be zero-NPV events. It is as if the bank has issued a guarantee. The value of the revolver is the value of the guarantee plus the value of the risk-free drawing. If one defines the “fees” on the revolver to include any spread above the “risk free rate” (generally LIBOR), then the actual drawing has no economic value. It is the right to issue a risk free instrument at the market rate for risk free instruments. After it is drawn, its value goes up by the face amount of the drawing. A key point is that repaying a commitment does not extinguish the obligation for the lender as the borrower could draw again during the lifetime of the commitment.

Thus the value of a revolver is the present discounted value of the loss should the borrower default (a negative number) plus the present discounted value of the fees against the facility (a positive number) plus the face amount of any current drawn amount. Future variation in drawings does not impact the risk of the commitment except to the extent that it causes one to revise expectations of usage given default. Future variation in drawings does not impact the payoff of the commitment except to the extent that it leads one to revise expectations of future fee income.

## Conclusion

Viewing the liabilities of a corporation, including its equity, as derivatives on the underlying firm value provides a powerful framework for determining the market value of those liabilities that do not have current secondary market prices.

In particular, one can combine the estimated default frequency for a firm, derived from its equity value and volatility, with market wide information on bond prices to obtain high quality estimates of pure credit spreads. These in turn can be used with sector wide bond prices to determine the market's view of expected recovery in default as a function of seniority and structure.

This combination of equity and bond price information explains 80%-90% of the cross-sectional variation in reported bond prices, and tracks the evolution of individual securities' prices accurately over time.

Further, the structural approach to default risk provides an empirically implementable framework for the valuation of the common options embedded in loan and bond prices, such as prepayment and revolver usage. Applied to a sample of actual secondary market loan prices, it accords well with the data.

The market valuation methodologies described here are not theoretical; they are empirically implemented and are being used in practice by banks to value corporate liabilities and, notably, bank loans. In contrast to alternative approaches that rely heavily upon accounting values and/or implied returns on allocated book capital, the methods utilized here provide accurate and essentially real time concordance with pricing in secondary markets. These developments have both immediate and far-reaching implications for the origination, intermediation and management of credit risk.



## Bibliography

Black, Fischer and John Cox [1976]: "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", *Journal of Finance*, 31, pp. 351-367.

Black, Fischer and Myron Scholes [1973]: "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81, pp. 637-659.

Bohn, Jeffrey R. [2000a]: "A Survey of Contingent-Claims Approaches to Risky Debt Valuation", *Journal of Risk Finance*, 1(3), pp. 53-78.

Bohn, Jeffrey R. [2000b]: "An Empirical assessment of a Simple Contingent-Claims Model for the Valuation of Risky Debt", *Journal of Risk Finance*, 1(4), pp. 55-77.

Collin-Dufresne, Pierre and Bruno Solnik [2001]: "On the Term Structure of Default Premia in the Swap and LIBOR Markets," *Journal of Finance*, 56(3), 1095-1115.

Crouhy, Michel, Dan Galai and Robert Mark [2001]: "*Risk Management*."

Ingersoll, Jonathon [1977a]: "A Contingent-Claims Valuation of Convertible Securities", *Journal of Financial Economics*, 4, pp. 289-321.

Ingersoll, Jonathon [1977b]: "An Examination of Corporate Call Policies on Convertible Securities", *Journal of Finance*, 32, pp. 463-478.

Kealhofer, Stephen, Sherry Kwok and Wenlong Weng [1998]: "Uses and Abuses of Bond Default Rates", *CreditMetrics Monitor*.

Kealhofer, Stephen [2002]: "Quantifying Credit Risk", forthcoming in *Financial Analysts Journal*

Merton, Robert C. [1973]: "Theory of Rational Option Pricing", *Bell Journal of Economics*, Spring 1973, pp. 141-183.

Merton, Robert C. [1974]: "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance*, 29, pp. 449-470.

Miller, Ross [1998], "Refining Ratings", *RISK*, August.

Vasicek, Oldrich A. [1984]: "Credit Valuation", unpublished, KMV.

Vasicek, Oldrich A. [1998]: "Credit Valuation", *NetExposure*, Issue 1, 1997; reprinted on CD in *Derivatives: Theory and Practice of Financial Engineering*, P. Wilmott, J. Wiley & Sons, London, Great Britain.

Vasicek, Oldrich A. [2001]: "Actual and Risk-Neutral Probabilities of Default", unpublished, KMV.